UNIVERSITY OF Southampton
School of Social Sciences

Economics Division
University of Southampton
Southampton SO17 1BJ, UK

Discussion Papers in
Economics and Econometrics

Endogenous ambiguity in cheap talk

(Christian Kellner (University of Southampton) Mark Thordal-Le Quement (University of Bonn)

No. 1701

This paper is available on our website
http://www.southampton.ac.uk/socsci/economics/research/papers

ISSN 0966-4246

Endogenous ambiguity in cheap talk

Christian Kellner and Mark Thordal-Le Quement*

Feb 2017

Abstract

This paper proposes a model of ambiguous language. We consider a cheap talk game in which a (possibly ambiguity averse) sender who faces an ambiguity averse receiver is able to randomize according to unknown probabilities. We show that under fairly general conditions, for any standard influential communication equilibrium there exists a Pareto-dominant equilibrium featuring an ambiguous (i.e. Ellsbergian) communication strategy. Ambiguity, by triggering worst-case decision-making by the receiver, shifts the latter's response to information towards the sender's ideal action, thus encouraging finer information transmission.

Keywords: cheap talk, ambiguity.

JEL classification: D81, D83.

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*Department of Economics, University of Bonn, Institute of Microeconomics. We thank Eric Danan, Sidartha Gordon, Martin Hellwig, Jean-Philippe Lefort, Peter Klibanoff, Daniel Krähmer, Stefan Lauermann, Anders Poulsen, Frank Riedel, Gerhard Riener, Robert Sugden, Dezsö Szalay as well as seminar and conference audiences in Bonn, MPI Bonn, SAET 2015 in Cambridge, Workshops on Ambiguity in Bielefeld and Paris, EEA 2015 in Mannheim, ICGT 2015 in Stony Brook, ESWC 2015 in Montreal, GREQAM Aix-Marseille, LEDA Paris-Dauphine, THEMA Cergy-Pontoise, Namur, Southampton, Düsseldorf, East-Anglia.

1 Introduction

Ambiguous language is a recurrent feature of economic and political communication. The term *Fedspeak* refers to the cryptic language used by chairmen of the Federal Reserve Board in stating their economic prognosis or their intended course of action. Politicians as well as experts similarly often use language that is difficult to parse and gives rise to different interpretations. On the face of it, this is puzzling because such language appears to gratuitously decrease the precision of transmitted information as well as impose costs of interpretation on the receiver. In the standard cheap talk game à la Crawford and Sobel (1982) that we consider, we find that ambiguous language on the contrary plays a positive role. It improves information transmission and increases the payoffs achievable by both parties.

An informed sender (S) faces an uninformed receiver (R) and S is known to favour a higher action than R for any realization of the state. R is ambiguity averse and applies Max-Min expected utility in the presence of ambiguity. We show that under fairly general conditions, S and R can both benefit from the use of an ambiguous communication strategy according to which S voluntarily conditions her messages on the privately observed realization of a draw from an urn containing a distribution of ball colors unknown to R (a so-called Ellsberg urn). We find that for any influential communication equilibrium featuring a standard communication strategy, there exists an equilibrium featuring an ambiguous communication strategy which strictly Pareto-dominates it. Ambiguity mitigates conflict by shifting upwards R's response to information and thus renders finer information transmission incentive compatible. S gains because she effectively faces a less misaligned receiver. R also benefits because her suboptimal response to information (from an ex ante perspective) is more than compensated by the availability of more information.

We now provide more detail regarding the underlying mechanism. In the classical Crawford and Sobel (1982) model, preference misalignment causes noisy communication. Any equilibrium outcome can be implemented via a so-called partitional equilibrium. The state space is cut up into a set of adjacent intervals $1, ..., N$ and S simply reveals the interval in which the state is located by sending m_i when the state is in interval i. If preference misalignment is reduced, the finest equilibrium partitioning becomes finer and yields a higher expected payoff for both parties. This paper identifies a new type of communication strategy which, by creating local Knightean uncertainty, effectively allows R to commit to acting as if her preferences were less misaligned than they are. Such a strategy is constructed as follows. Given a set of standard intervals $1, ..., N$, subdivide every standard interval i into two adjacent subintervals i_- and i_+ mapping into two different messages m_i^A and m_i^B. For each standard interval i, let S randomize between one pure messaging rule mapping from $\{i_-, i_+\}$ into $\{m_i^A, m_i^B\}$ and the reciprocal rule conditional on the privately observed color (red or non-red) of the ball drawn from the available Ellsberg urn. Given this strategy, upon observing m_i^A and m_i^B, R is Knighteanly uncertain as to whether the state is more likely situated in i_- or i_+. Being ambiguity averse, R acts so as to hedge herself against ambiguity and follows the Max-Min decision rule. This involves evaluating every action according to its lowest (worst case) expected utility under all possible priors (i.e. all possible compositions of the urn) and picking the action that maximizes the thus constructed objective function. The key mechanism is that if the left subinterval i_- is significantly larger than the right subinterval i_+, so that the state is ex ante much more likely to be situated in i_- than in i_+, R acts as if subjectively overweighting the event that the state is located in the right subinterval i_+. The reason is that, being driven by worse-case thinking, R evaluates all low actions as if certain that the state is located in i_+, no matter how unlikely this event. As a result, R takes a higher action than the expected utility maximizing action conditional on the information that the state is located in the standard interval i.

For the sake of concreteness, consider the following example of an ambiguous strategy. S privately observes a state $\omega \in [0, 1]$, drawn from a commonly known distribution, which is S's assessment of growth prospects for the economy. The color of the ball drawn from the Ellsberg urn is S's mood whose distribution is unknown to R. Divide the $[0, 1]$ segment into four adjacent subintervals $1, ..., 4$. If S draws a red ball she uses the communication strategy {*mediocre,poor,sustained,strong*}, where the ith message in this ordered set is sent on the ith subinterval. If S instead draws a black ball she instead uses the communication strategy {*poor,mediocre,strong,sustained*}. Hence, in terms of the explanation in the previous paragraph the first two subintervals together constitute one standard interval and the next two subintervals constitute another standard interval. Conditional on S randomizing in such a way, R's updating exhibits the following characteristics. If the received message m belongs to {*poor,mediocre*} then R now knows that ω is located one of the first two subintervals but is Knighteanly uncertain as to whether it is actually located in the first or the second. If on the other hand m belongs to {*sustained,strong*}, R now knows that ω is located in the third or fourth subinterval but is Knighteanly uncertain as to whether it is actually located in the third or the fourth. If the second subinterval is substantially smaller than the first, R's (identical) best response to *poor* or *mediocre* will be higher than her best response to the information that ω is located in one of the first two subintervals. An identical argument holds for the two remaining messages and subintervals.

We contribute to the literature by studying Ellsbergian strategies within the classical Crawford and Sobel (1982) cheap talk game (CS in what follows). In so-doing, we build on Azrieli and Teper (2011), Bade (2010) as well as Riedel and Sass (2011) who introduce the concept of ambiguous strategies and equilibrium under such strategies[1] but do not apply it to cheap talk. Bose and Renou (2014) states a revelation principle for environments where the principal can use an Ellsbergian device to garble his messages to agents. The authors furthermore obtain a full surplus extraction result for the monopolistic screening problem. While the ambiguous communication strategies that we introduce build on Bose and Renou (2014), main differences are that we assume communication without commitment and that our focus is on welfare implications within the Crawford and Sobel (1982) game.

Kellner and Le Quement (2014) examines cheap talk with an ambiguous prior distribution of the state. The latter and the present paper share the feature that S-optimal equilibria exhibit a communication strategy that is not describable as a standard partitional strategy but rather as a randomization across such strategies. Randomization follows a known distribution in Kellner and Le Quement (2014) and instead an ambiguous distribution in the present paper. While randomization serves to hedge S against exogenous ambiguity in Kellner and Le Quement (2014), it serves to induce R to hedge against

[1]See also earlier work by Lo (1996) and Klibanoff (1996) on equilibrium in ambiguous beliefs.

endogenous ambiguity in the present paper.

Board, Blume and Kawamura (2007) analyze vagueness generated by a noisy communication channel that causes sent messages to give rise to a distribution over observed messages. The authors construct equilibria that are more informative than the most informative equilibria of the noiseless model. Equilibria are partitional and R's expectation of S's type given a message is a weighted average of the conditional expectation given the implied interval in the absence of transmission error and the expectation conditional on an error. As a result, R's expectation given a message indicating a low interval is distorted upwards towards the ex ante mean. For low values of the state, the de facto preference misalignment of S and R is thus reduced compared to the noiseless setup. The key similarity between this paper and ours resides in the fact that noise helps R de facto commit to taking a higher action in response to information than she would in the absence of noise. In our paper, noise however has a different origin (it is voluntarily inserted by S), it comes in a different form (it is ambiguous) and it affect's R's behavior via a different motive (hedging against ambiguity).

Our results also relate to Chen and Gordon (2015). The authors define a nestedness relation between sender-receiver games, one game being nested within the other if the optimal actions of players are closer in the first. They identify general conditions under which the nested game yields more information transmission and higher expected payoffs for S and R. Roughly speaking, our cheap talk game with Ellsbergian communication strategies can be interpreted as being (endogenously) nested within the same game with standard communication strategies.

This paper is organized as follows. Section 2 introduces the model. Section 3 states central (and with one minor exception) preexisting results for the standard case. Section 4 analyzes the general Ellsbergian model and contains our main result concerning the existence of Pareto-improving Ellsbergian equilibria. Section 5 examines the special case of the Uniform-Quadratic setup and studies specific analytically tractable subclasses of Ellsbergian communication strategies, with an eye to examining questions left unanswered within the general setup. Section 6 discusses the robustness of our results to key restrictive assumptions.

2 Model

There are two players, a sender S and a receiver R. The state ω is privately known to S and drawn from a commonly known distribution endowed with the continuously differentiable cdf F and density f on the support $[0,1]$. S privately draws a ball from an Ellsberg urn containing balls of n different colors numbered 1 to n. Let ρ_i denote the proportion of color i balls. The vector $\rho = (\rho_1, ..., \rho_n)$ is Knighteanly unknown to S and R. Letting Δ^n denote the set of all vectors $\rho = (\rho_1, ..., \rho_n)$ satisfying $\sum_{i=1}^{n} \rho_i = 1$, the Ellsberg urn is maximally ambiguous in the sense that any ρ in Δ^n is considered possible. Let θ be a random variable taking value θ_i if the drawn ball has color i. The timing of the game is as follows. S observes ω and θ. S picks a message $m \in M$, where M is a rich message space with cardinality $|M|$. R picks an action $a \in \mathbb{R}$ after observing m. Given a and ω, the utility function of $J \in \{S, R\}$ is denoted $U^J(a,\omega)$ and

$$U^S(a,\omega) = G(\omega + \beta(\omega) - a)\,;\; U^R(a,\omega) = G(\omega - a)\,.$$

The function $G(x)$ is a twice differentiable, concave and single peaked function of x with a peak a $x = 0$. There is an $\varepsilon > 0$ such that $\beta(\omega) > \varepsilon$ for any $\omega \in [0,1]$. For two bias function $\beta(\omega)$ and $\widehat{\beta}(\omega)$ such that $\widehat{\beta}(\omega) > \beta(\omega)\ \forall\omega$, we write $\widehat{\beta} > \beta$. For given β and $\varepsilon > 0$, let $\beta + \varepsilon$ denote $\widehat{\beta}$ such that $\widehat{\beta}(\omega) = \beta(\omega) + \varepsilon\ \forall\omega$. Letting subscripts denote partial derivatives, our setup implies that for any $J \in \{R, S\}$, $U_1^J = 0$ for some a, $U_{11}^J < 0$ and $U_{12}^J > 0$ as in CS. Both S and R are ambiguity averse and apply the Max-Min decision rule (Gilboa (1987), Gilboa and Schmeidler (1989)).

A standard communication strategy is given by a family $q(.\,|\omega)$, $\omega \in [0,1]$, of distributions. Such a family defines a distribution over M for each value of ω and is thus a mapping $[0,1] \to \Delta^{|M|}$, where $\Delta^{|M|}$ is the set of distributions over M. An Ellsbergian communication strategy is given by a vector of standard communication strategies denoted $(q_1(.\,|\omega), ..., q_n(.\,|\omega))$. S plays such a strategy by conditioning her choice of q_i on the value of θ, more precisely by following q_i if $\theta = \theta_i$. A (mixed) strategy of R specifies a distribution $\delta(.\,|m)$ over pure actions for any $m \in M$. Letting $\Delta^{\mathbb{R}}$ denote the set of distributions over $\mathbb{R}$, a strategy of R is a mapping $M \to \Delta^{\mathbb{R}}$.

We define an equilibrium concept that is an analogue of Perfect Bayesian equilibrium for the case where S can use an Ellsbergian strategy. Our equivalents of the sequential rationality and consistent beliefs conditions reduce to the following requirements here. The strategy of S is sequentially rational, given the strategy of R. R applies the Max-Min rule conditional on updated beliefs. As to belief formation, R performs prior by prior Bayesian updating conditional on knowledge of S's equilibrium strategy[2]. Note that S faces no ambiguity at any information set where she is called upon to act. This is true because S observes θ before choosing a message. When S chooses her message after observing ω and θ, there is thus no mixed strategy that does better than all pure strategies in its support, which might be the case if she faced ambiguity. The incentive compatibility conditions for S are thus identical to those defined in the classical CS game without Ellsbergian randomization.

Formally, a strategy profile $\left(q_1(m\,|\omega\,), ..., q_n(m\,|\omega\,)\right), \delta(a\,|m\,)$ and a belief system constitute an equilibrium if the following conditions hold. First, $\int_M q_i(m\,|\omega\,)dm = 1\ \forall\,(\omega, i) \in [0,1] \times \{1, ..., n\}$, where any m^* in the support of $q_i(m\,|\omega\,)$ solves

$$\max_{m\in M} \int_{a\in\mathbb{R}} U^S(a,\omega)\delta(a\,|m\,)da. \tag{1}$$

Second, for each m, δ^* solves

$$\max_{\delta\in\Delta^{\mathbb{R}}}\min_{\rho\in\Delta^n} \int_0^1 \left(\int_{a\in\mathbb{R}} U^R(a,\omega)\delta(a\,|m\,)da\right) p(\omega\,|m,\rho\,)d\omega, \tag{2}$$

where

$$p(\omega\,|m,\rho\,) = \frac{\sum\limits_{i\in\{1,...,n\}} p(\theta_i\,|\rho\,)q_i(m\,|\omega\,)f(\omega)}{\int_0^1 \sum\limits_{i\in\{1,...,n\}} p(\theta_i\,|\rho\,)q_i(m\,|t\,)f(t)dt}$$

is R's posterior belief given message m and urn composition ρ.

We follow Sobel (2013) in distinguishing between informative, influential and payoff-relevant communication. Communication is informative if it affects beliefs, i.e. if $p(.\,|m,\rho\,)$

[2] A consensus has yet to emerge on the right modelling of updating of ambiguity averse preferences. We refer to Siniscalchi (2011) and Hanany and Klibanoff (2007, 2009) for a discussion of this issue. A dynamic Ellsberg experiment by Dominiak, Dürsch and Lefort (2012) finds that more subjects satisfy consequentialism than dynamic consistency.

is not constant across messages for all ρ's. Communication is influential if it affects actions, i.e. if $\delta(.\,|m)$ is not constant on the equilibrium path. Communication is payoff-relevant if at least one agent's ex ante expected payoff differs from that implied by R's ex ante payoff maximizing action. Denote by respectively $\pi^S\left(\beta, \widetilde{E}\right)$ and $\pi^R\left(\widetilde{E}\right)$ the (ex ante) expected payoff of S and R given the decision rule implemented in equilibrium $\widetilde{E}$. The notion of ex ante expected payoffs is unproblematic as we examine equilibria featuring no ex ante uncertainty regarding the implemented decision rule.

3 The standard case

Our utility functions are a special case of those assumed in CS, which allows us to directly invoke existing comparative static results. Consider two sender utility functions featuring respectively β and β' with $\beta' > \beta$. As shown in the following Lemma, these can be generated from a common function $U^S(a, \omega, b)$ satisfying the assumptions made in sections 2 and 5 of CS, these assumptions being that $U^S(a, \omega, b)$ is such that b is a scalar parameter measuring interest misalignment, $U^S_{13} \geq 0$ everywhere and $U^S(a, \omega, 0) = U^R(a, \omega)$.

Lemma 1 *Consider G, β and β' such that $\beta' > \beta$. There is a function $U^S(a, \omega, b)$ such that a) $U^S(a, \omega, 1) = G(\omega + \beta(\omega) - a)$, b) $U^S(a, \omega, 2) = G\left(\omega + \beta'(\omega) - a\right)$, c) $U^S(a, \omega, 0) = G(\omega - a)$, d) $U^S_{13}(a, \omega, b)$ is strictly positive everywhere.*

Proof: see Appendix A.

We know from CS that absent Ellsbergian strategies, any equilibrium is equivalent to an equilibrium featuring a standard partitional communication strategy. Let there be messages labelled $\{m_i\}_{i=0}^{N-1}$. A standard partitional communication strategy is described by a vector of thresholds $t_0 = 0 < t_1 < ... < t_N = 1$ such that sender type 0 sends m_0 and all types in $(t_i, t_{i+1}]$ send m_i $\forall i$. We call an equilibrium featuring such a strategy a standard equilibrium and call N its fineness. We call a standard equilibrium non-degenerate if each action is induced by a set of types with positive measure, i.e. if $0 < t_1(\beta, N) < ... < t_{N-1}(\beta, N) < 1$. Denote by $a^*_{ne}(t_{i-1}, t_i)$ R's optimal action given $\omega \in (t_{i-1}, t_i]$. The profile of thresholds $\{t_i\}_{i=1}^{N-1}$ constitutes a standard equilibrium iff

$$U^S(a_{ne}^*(t_{i-1},t_i),t_i) = U^S(a_{ne}^*(t_i,t_{i+1}),t_i),\ i = 1,...,N-1. \tag{3}$$

Randomization by S is thus not useful in the standard case. For every equilibrium involving randomization (for example by S conditioning her messaging on some payoff-irrelevant private signal), there is indeed a standard partitional equilibrium implementing the same decision rule.

CS states the following monotonicity condition **M:** If t and $\widetilde{t}$ are two solutions of (3) with $t_0 = \widetilde{t}_0$ and $\widetilde{t}_1 > t_1$, then $\widetilde{t}_i > t_i$, for any $i \geq 2$. CS and Szalay (2012) provide different sufficient conditions for **M**. The condition is known to hold if the state is uniformly distributed, $G(x) = -x^2$ and $\beta(\omega) = b$ for all ω (so-called Uniform-Quadratic specification).

Assumption 1 *Condition **M** holds.*

We next recall a set of basic properties of the model which we directly build upon in our analysis of the Ellsbergian case. Given N, let $\Gamma(N)$ denote the set of βs such that there exists an N-intervals equilibrium.

Proposition 1 ***Aspects of the Crawford & Sobel (1982) characterization.***

Assume that S is restricted to using standard strategies.

1. Given β, there is a finite $\overline{N}(\beta)$ such that for all $N \leq \overline{N}(\beta)$ (for all $N > \overline{N}(\beta)$), there is a unique (there is no) N-intervals equilibrium. We call the unique N-intervals equilibrium $E(\beta, N)$ and denote its threshold profile by $\{t_i(\beta,N)\}_{i=1}^{N-1}$.

2. $\overline{N}(\beta) \geq \overline{N}(\beta')$ if $\beta' > \beta$.

3. $\pi^R(E(\beta,N)) > \pi^R(E(\beta',N))$ for $\beta < \beta' \in \Gamma(N)$.

4. $\pi^S(\beta, E(\beta, N-1)) < \pi^S(\beta, E(\beta, N))$ and $\pi^R(E(\beta, N-1)) < \pi^R(E(\beta,N))$ for $\beta \in \Gamma(N)$.

Point 2 states that a higher bias leads to a weakly lower maximal number of equilibrium intervals. Point 3 states that given N, R's expected payoff decreases as bias increases. Point 4 states that given β, both S and R favor equilibria with more intervals. We now show that given a fixed number N of equilibrium intervals, a less biased S obtains a higher expected payoff. This result does not appear in CS and is an equivalent of Point 3 for S.

Lemma 2 *$\pi^S(\beta, E(\beta, N)) > \pi^S(\beta+\varepsilon, E(\beta+\varepsilon, N))$ for β, ε such that $\beta, \beta+\varepsilon \in \Gamma(N)$.*

Proof: see Appendix A.

4 The Ellsbergian case

This section is organized as follows. We first introduce the class of Ellsbergian partitional communication strategies and present equilibrium conditions for corresponding equilibria. We then provide our main result concerning the existence of Pareto-improving Ellsbergian equilibria. We finish by noting the existence of a new type of (Ellsbergian) babbling equilibrium.

Simple Ellsberg randomization, defined below, is a key building block of the Ellsbergian partitional communication strategies that we shall focus on.

Definition 1 ***Simple Ellsberg randomization φ***

Let $(\underline{\omega}, \overline{\omega}] \subseteq [0,1]$ and $c \in (\underline{\omega}, \overline{\omega}]$. The simple Ellsberg randomization $\varphi(\underline{\omega}, \overline{\omega}, c, m, m')$ is defined as follows. Given $\theta = \theta_1$, send m with probability 1 if $\omega \in (\underline{\omega}, c)$ and m' with probability 1 if $\omega \in [c, \overline{\omega}]$. If $\theta \neq \theta_1$, send m' with probability 1 if $\omega \in (\underline{\omega}, c)$ and m with probability 1 if $\omega \in [c, \overline{\omega}]$.

A simple Ellsberg randomization is thus a randomization with unknown probabilities over two reciprocal partitional strategies on $(\underline{\omega}, \overline{\omega}]$ which both partition $(\underline{\omega}, \overline{\omega}]$ into two subintervals $(\underline{\omega}, c)$ and $[c, \overline{\omega}]$. One partitional strategy sends m in the lower subinterval and m' in the upper subinterval while the other partitional strategy does the opposite. We now introduce the Ellsbergian communication strategy that shall be the focus of our analysis.

Definition 2 ***Ellsbergian partitional communication strategy***

Let there be two profiles of thresholds $t_0 = 0 < t_1 < ... < t_{N-1} < t_N = 1$ and $\{c_i\}_{i=0}^{N-1}$ such that $c_i \in (t_i, t_{i+1}]$, $i = 0, ..., N-1$. If $\omega \in (t_i, t_{i+1}]$, S applies $\varphi\left(t_i, t_{i+1}, c_i, m_i^A, m_i^B\right)$, $i = 0, ..., N-1$. An Ellsbergian partitional communication strategy is thus summarized by $\left\{\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right\}$.

An Ellsbergian partitional communication strategy simply adds Ellsbergian randomization to a standard partitional strategy and involves the following simple two-steps procedure. S first determines the interval $(t_i, t_{i+1}]$ in which ω is located. She then applies the randomization $\varphi\left(t_i, t_{i+1}, c_i, m_i^A, m_i^B\right)$. For a given Ellsbergian partitional strategy featuring $\{t_i\}_{i=1}^{N-1}$, we still refer to N as the number of intervals (or fineness). We call an equilibrium featuring an Ellsbergian partitional communication strategy an Ellsbergian partitional equilibrium.

Denote by $a_{ne}^*(I)$ R's best response to $\omega \in I$, where I is an interval of $[0,1]$. We assume the following.

Assumption 2 *Let $0 \leq \underline{\omega} < \overline{\omega} \leq 1$ and $c \in (\underline{\omega}, \overline{\omega}]$. Let $I_1 = (\underline{\omega}, c)$ and $I_2 = [c, \overline{\omega}]$. For any $i, j \in \{1,2\}$ and $j \neq i$,*

$$E\left[U^R(a_{ne}^*(I_i), \omega)\,|\omega \in I_i\right] > E\left[U^R(a_{ne}^*(I_i), \omega)\,|\omega \in I_j\right]. \tag{4}$$

Consider two adjacent intervals I_i and I_j. The expected payoff of action $a_{ne}^*(I_i)$ conditional on $\omega \in I_i$ should thus be strictly larger than conditional on $\omega \in I_j$. Technically speaking, the assumption ensures well-behaved expected utility curves which yields a simple characterization of R's best responses as given in the next Lemma. We know that Assumption 2 holds given the assumed single peaked and concave utility function if the state is uniformly distributed, implying that it is satisfied in the Uniform-Quadratic specification of the model.

Lemma 3 *Given the Ellsbergian communication strategy $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$, R's best response to m_i^A and m_i^B is identical. Denote it by $a_e^*(t_i, t_{i+1}, c_i)$.*

a) a_e^* satisfies:

$$E\left[U^R(a_e^*, \omega)\,|\omega \in (t_i, c_i)\right] = E\left[U^R(a_e^*, \omega)\,|\omega \in [c_i, t_{i+1}]\right]$$

b) $a_e^*(t_i, t_{i+1}, c_i)$ is a continuous and strictly increasing function of c_i and

$$a_e^*(t_i, t_{i+1}, t_i) < a_{ne}^*(t_i, t_{i+1}) < a_e^*(t_i, t_{i+1}, t_{i+1}). \tag{5}$$

Proof: see Appendix B.

There are two aspects. The first is how to identify the best response of R to m_i^A and m_i^B (Point a.) and the second is how a shift to the right in c_i increases this best response (Point b.). We give some intuition for both in what follows.

Upon observing a message (say m_i^A), for each possible value of ρ_1 the receiver updates beliefs separately and constructs a corresponding expected utility curve. Whatever the ρ_1 assumed, R now knows that $\omega \in (t_i, t_{i+1}]$. The particular value of ρ_1 assumed will affect the weight attributed to the subintervals (t_i, c_i) and $[c_i, t_{i+1}]$. If for example $\rho_1 = 1$, then observing m_i^A is equivalent to learning that $\omega \in (t_i, c_i)$. If instead $\rho_1 = 0$, then observing m_i^A is equivalent to learning that $\omega \in [c_i, t_{i+1}]$. If $\rho_1 \in (0, 1)$, then m_i^A does not allow to pin down with certainty the subinterval in which ω is situated but will (except in knife edge cases) imply a conditional state distribution with support over both subintervals (t_i, c_i) and $[c_i, t_{i+1}]$. Assume for example a uniform distribution of ω and $c_i = \frac{t_i + t_{i+1}}{2}$. In such a case, given $\rho_1 = \frac{2}{3}$ message m_i^A would be equivalent to learning that with probability $\frac{2}{3}$, the state is distributed uniformly on (t_i, c_i) while with probability $\frac{1}{3}$ it is distributed uniformly on $[c_i, t_{i+1}]$.

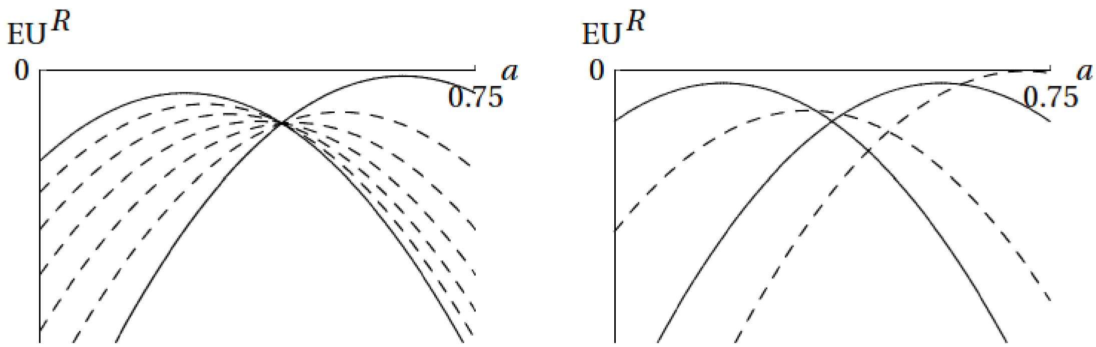
Graphically, the Max-Min best response of R to a message (say m_i^A) is obtained as follows. First, consider the set of expected utility curves corresponding to different $\rho_1' s$, which is constructed as explained above. Each of these curves is a concave and single peaked function of R's action a. Note that for any given a, the expected payoff given $\rho_1 \in (0, 1)$ is strictly between the expected payoffs corresponding to respectively $\rho_1 = 0$ and $\rho_1 = 1$. Second, construct the lower envelope by simply attaching to each a the lowest possible expected payoff across $\rho_1' s$. A single crossing condition holds. There is a threshold value of a (call this $\widetilde{a}$) such that the lowest expected payoff across $\rho_1' s$ corresponds to $\rho_1 = 0$ for $a < \widetilde{a}$ and instead to $\rho_1 = 1$ for $a > \widetilde{a}$. Furthermore, the $\rho_1 = 0$ curve is increasing in a below $\widetilde{a}$ while the $\rho_1 = 1$ curve is decreasing in a above $\widetilde{a}$. The constructed lower envelope is thus maximized at the intersection point of the $\rho_1 = 0$ and $\rho_1 = 1$ curves, implying that R's Max-Min action after m_i^A is $\widetilde{a}$.

Now, consider R's best response to m_i^B. This corresponds again to the intersection of the $\rho_1 = 0$ and the $\rho_1 = 1$ curves, but the curve corresponding to $\rho_1 = 0$ given m_i^A now corresponds to $\rho_1 = 1$ given m_i^B. Similarly, the curve corresponding to $\rho_1 = 1$ given m_i^A

now corresponds to $\rho_1 = 0$ given m_i^B. It follows that R's Max-Min best response to m_i^B is the same as her best response to m_i^A. Note the following two other properties of R's Max-Min best response to m_i^A and m_i^B. First, it fully hedges R against ambiguity by equalizing R's expected payoff under all possible values of ρ_1. Second, Assumption 2 implies that the Max-Min best response lies strictly between the peaks $a_{ne}^*(t_i, c_i)$ and $a_{ne}^*(c_i, t_{i+1})$.

We now discuss how a shift to the right in c_i increases R's best response to m_i^A and m_i^B. As c_i increases, both $E\left[U^R(a,\omega)\,|\omega \in (t_i, c_i)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c_i, t_{i+1}]\right]$ shift to the right so that their intersection $a_e^*(t_i, t_{i+1}, c_i)$ naturally also shifts to the right. To see that S can use Ellsbergian randomization to trigger a best response to m_i^A and m_i^B that is higher than R's standard best response $a_e^*(t_i, t_{i+1})$ to the information that $\omega \in (t_i, t_{i+1}]$, simply note the following. The action $a_e^*(t_i, t_{i+1})$ is the one that maximizes the expected utility function $E\left[U^R(a,\omega)\,|\omega \in (t_i, t_{i+1})\right]$ while the Max-Min action $a_e^*(t_i, t_{i+1}, t_{i+1})$ is the action at which $E\left[U^R(a,\omega)\,|\omega \in (t_i, t_{i+1})\right]$ and $E\left[U^R(a,\omega)\,|\omega = t_{i+1}\right]$ intersect, this action being located to the right of the action at which $E\left[U^R(a,\omega)\,|\omega \in (t_i, t_{i+1})\right]$ peaks, as ensured by Assumption 2. The same argument shows that $a_e^*(t_i, t_{i+1}, t_i) < a_e^*(t_i, t_{i+1})$. By continuity of $a_e^*(t_i, t_{i+1}, c_i)$ in c_i, it follows that S can trigger any action between $a_e^*(t_i, t_{i+1}, t_i)$ and $a_e^*(t_i, t_{i+1}, t_{i+1})$ by adjusting c_i.

Figures 1 and 2 exemplify the above. We assume a uniform distribution of the state and $U^R(a,\omega) = (\omega - a)^2$. We set $t_i = 0$, $t_{i+1} = .75$. In Figure 1, we set $c_i = .5$. Continuous curves correspond to $E\left[U^R(a,\omega)\,\middle|m_i^A, \rho_1 = 1\right]$ and $E\left[U^R(a,\omega)\,\middle|m_i^A, \rho_1 = 0\right]$ while remaining dashed curves correspond to interior values of ρ_1. In Figure 2, we consider two possible values of c_i, respectively .375 and .6. Continuous curves correspond to $E\left[U^R(a,\omega)\,\middle|m_i^A, \rho_1 = 1\right]$ and $E\left[U^R(a,\omega)\,\middle|m_i^A, \rho_1 = 0\right]$ for $c_i = .375$. Dashed curves are equivalents for the case of $c_i = .6$.



Figures 1 and 2

We now characterize incentive conditions for S. The following Lemma shows that equilibrium conditions have a similar form as in CS.

Lemma 4 *An equilibrium featuring the Ellsbergian communication strategy* $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ *exists if and only if*

$$U^S(a_e^*(t_{i-1}, t_i, c_{i-1}), t_i) = U^S(a_e^*(t_i, t_{i+1}, c_i), t_i),\ i = 1, ..., N-1. \tag{6}$$

Proof: $\forall i \in \{1, ..., N-1\}$, m_i^A and m_i^B trigger an identical best response, so S is indifferent between m_i^A and m_i^B for any $\omega \in (t_i, t_{i+1}]$. We thus only need to consider deviations across messages carrying different subscripts. Condition (6) ensures that $\forall (t_i, t_{i+1}]$, S weakly prefers any element of $\{m_i^A, m_i^B\}$ to any other equilibrium message. ■

Note that (6) is identical to the standard equilibrium condition (3), except R's best response is now $a_e^*(t_i, t_{i+1}, c_i)$ instead of $a_{ne}^*(t_i, t_{i+1})$.

As a preliminary comment to our main result appearing below, note that on the equilibrium path of a partitional Ellsbergian equilibrium, the expected payoff of S and R is independent of the actual composition of the urn. This is true for two reasons which are both contained in Lemma 3. First, randomization conditional on θ involves messages which trigger identical actions by R. Second, R's expected payoff conditional on receiving m_i^A or m_i^B is the same for any $i = 1, ..., N$. Hence, the Max-Min payoff of each agent at any stage is equal to her expected payoff assuming for example that $\rho_1 = .5$.

Proposition 2 *Given β and $N \geq 2$, if the standard equilibrium $E(\beta, N)$ exists and is non-degenerate, there is an $\overline{\varepsilon} > 0$ such that for any $\varepsilon \leq \overline{\varepsilon}$, there exists an Ellsbergian partitional equilibrium $\widetilde{E}$ summarized by $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ such that:*

a) $t_i = t_i(\beta - \varepsilon, N)$, $i = 1, ..., N-1$.

b) $a_e^*(t_i, t_{i+1}, c_i) = a_{ne}^*(t_i, t_{i+1}) + \varepsilon$, $i = 0, ..., N-1$.

c) S's expected payoff in $\widetilde{E}$ is the same as the expected payoff that she obtains in the standard equilibrium $E(\beta - \varepsilon, N)$ if her bias is $\beta - \varepsilon$, which is strictly larger than her expected payoff in the standard equilibrium $E(\beta, N)$.

d) R's expected payoff in $\widetilde{E}$ is strictly larger than her expected payoff in the standard equilibrium $E(\beta, N)$.

Proof: See Appendix C.

If the standard equilibrium $E(\beta, N)$ exists and is non-degenerate, there thus exists an Ellsbergian equilibrium $\widetilde{E}$ that quasi-replicates the standard equilibrium $E(\beta - \varepsilon, N)$ of a game in which S is replaced by a sender with a lower bias $\beta - \varepsilon$. While $\widetilde{E}$ features the same profile of thresholds $\{t_i(\beta - \varepsilon, N)\}_{i=1}^{N-1}$ as $E(\beta - \varepsilon, N)$, R's action for each interval is shifted upwards given

$$a_e^*(t_i, t_{i+1}, c_i) = a_{ne}^*(t_i, t_{i+1}) + \varepsilon, \; i = 0, ..., N-1. \tag{7}$$

$\widetilde{E}$ is thus not outcome-equivalent to $E(\beta - \varepsilon, N)$. The two equilibria are however virtually payoff-equivalent for S in the sense that $\pi^S\left(\beta, \widetilde{E}\right)$ is equal to the payoff obtained in $E(\beta - \varepsilon, N)$ by a sender with bias $\beta - \varepsilon$. Given Lemma 2, this expected payoff is furthermore larger than $\pi^S(\beta, E(\beta, N))$.

As to R, transiting from $E(\beta, N)$ to $\widetilde{E}$ implies a trade-off. While Proposition 1 implies that she prefers the new threshold profile conditional on best responding to intervals, her response to intervals now shifts away from the optimal one (see (7)). An Envelope Theorem argument ensures that the trade-off is resolved positively for ε low enough. Given the first order conditions holding at $a_{ne}^*(t_i(\beta - \varepsilon, N), t_{i+1}(\beta - \varepsilon, N))$, the cost of marginally shifting a upwards at that point is 0. For ε small, R thus achieves a substantial gain at a negligible cost by transiting from the standard equilibrium $E(\beta, N)$ to the Ellsbergian equilibrium $\widetilde{E}$.

The result for S parallels Theorem 2, part ii), in Chen and Gordon (2015) which says that S prefers to face a receiver with more aligned interests. As a consequence of the use of Ellsbergian randomization and her hedging objective, R shifts upwards her best response to any given interval as compared to her expected utility best response, i.e. acts as if she were more aligned. The result for R parallels Theorem 3, part ii), in Chen and Gordon (2015) which says that R prefers to face the greatest equilibrium partition corresponding to a different R with more aligned interests than herself. The difference of our result is that in our setting, facing a more attractive partition comes at the cost of imperfectly responding to the transmitted information from an ex ante point of view.

The following Corollary follows from the combined application of Proposition 1 and Proposition 2. In what follows, we say that an equilibrium E strictly Pareto-dominates another equilibrium $\widetilde{E}$ if E ensures both S and R a strictly higher expected payoff than $\widetilde{E}$

Corollary 1 *If there exists a non-degenerate and influential standard equilibrium, there exists an Ellsbergian partitional equilibrium that strictly Pareto-dominates any standard equilibrium.*

Proof**:** We know from Proposition 1 that absent Ellsbergian strategies, S and R's strictly preferred equilibrium is the finest standard equilibrium $E\left(\beta, \overline{N}(\beta)\right)$ (assuming without loss of generality that it is non-degenerate). Proposition 2 shows that given β, there exists an Ellsbergian equilibrium ensuring S and R a strictly higher expected payoff than that obtained in $E\left(\beta, \overline{N}(\beta)\right)$.■

We conclude our main analysis with a remark on the role of communication in our setup. Sobel (2013) writes for the standard case: "*In order for communication to be payoff-relevant for R it must be both informative and influential.*" and "*Relative to babbling, payoff-relevant communication must increase R's expected utility but may make S worse off.*". We now show that both of these properties break down once allowing for Ellsbergian strategies. Equilibrium communication can be payoff-relevant without being either informative or influential and it can be payoff-relevant while making S better-off and R worse-off. Let us call *standard babbling equilibrium* an equilibrium in which communication is non-informative and S uses a standard communication strategy.

Proposition 3 *There exists an equilibrium featuring non-informative, non-influential and payoff-relevant communication ensuring S (resp. R) an ex ante payoff strictly larger (resp. smaller) than* *the payoff obtained in the standard babbling equilibrium.*

Proof: By Lemma 3.b), there exists $\bar{\varepsilon} > 0$ such that for any $\varepsilon \leq \bar{\varepsilon}$, one can find a $c \in [0,1]$ yielding $a_e^*(0,1,c) = a_{ne}^*(0,1) + \varepsilon$. Also, given that $\beta(\omega) > 0$ for any $\omega \in [0,1]$, there is a $\overline{\delta} > 0$ such that $\forall \delta \leq \overline{\delta}$,

$$\int_0^1 U^S \left(a_{ne}^*(0,1) + \delta, \omega\right) f(\omega) d\omega > \int_0^1 U^S \left(a_{ne}^*(0,1), \omega\right) f(\omega) d\omega.$$

As to R, note that she necessarily loses in ex ante terms whenever $a_e^*(0,1,c)$ shifts away from $a_{ne}^*(0,1)$.■

The above equilibrium does not feature different messages that generate different sets of posteriors. Communication is thus non-informative, implying that it is also non-influential. Communication however generates a set of posteriors which is different from R's unique prior and which leads R to pick an action $a_e^*(0,1,c)$ that is different from her ex ante optimal action $a_{ne}^*(0,1)$. R loses in the ex ante sense because $a_e^*(0,1,c)$ is higher than her ex ante optimal action. S conversely gains ex ante for the same reason as long as $a_e^*(0,1,c)$ is not excessively shifted to the right with respect to $a_{ne}^*(0,1)$.

We call this equilibrium an Ellsbergian babbling equilibrium. Note that this equilibrium could be eliminated if we added an extra participation constraint for R stating that she only listens to S if this increases her ex ante payoff[3]. The focus of our analysis is indeed on equilibria that satisfy such a constraint as they Pareto-improve on standard influential communication equilibria.

5 More results for the Uniform-Quadratic case

The following section considers the so-called Uniform-Quadratic setup. We focus on two subclasses of Ellsbergian partitional equilibria that are both intuitive and analytically tractable. The first involves equally sized intervals and implements the optimal decision rule of S conditional on an exogenous restriction on the number of equilibrium actions. The second class involves a maximal use of Ellsbergian randomization. Our

[3]This would amount to assuming that the decision maker is sophisticated in the sense that she anticipates her future preference reversal. See Siniscalchi (2011).

main findings are as follows. First, for sufficiently low bias, both classes typically contain equilibria featuring more intervals than the finest standard equilibrium as well as Pareto-dominating the latter. Second, payoff improvements achieved through Ellsbergian communication can be significant. Third, Ellsbergian communication can generate the possibility of influential communication.

Assume ω is uniformly distributed on $[0,1]$ and

$$U^S(\omega,a,b) = -(a-(\omega+b))^2,\; U^R(\omega,a) = -(a-\omega)^2.$$

CS shows the following results. Given $N \geq 2$, there is a maximal bias $b_{ne}(N) = \frac{1}{2N(N-1)}$ such that for any $b \leq b_{ne}(N)$, there exists a unique N-intervals standard equilibrium. An influential standard equilibrium thus exists if and only if $b \leq \frac{1}{4}$. Given b, there is a maximal intervals number $N_{ne}(b) = \left\langle \frac{1}{2b}\left(b+\sqrt{b\,(b+2)}\right)\right\rangle$ such that for any $N \leq N_{ne}(b)$, there exists a unique N-intervals standard equilibrium (where $\langle x\rangle$ denotes the highest integer smaller than x).

For purely didactic purposes, we briefly rederive key aspects of our central result (Proposition 2) in the Uniform-Quadratic setup. Note first that the best response characterized in Lemma 3 reads

$$a_e^*(t_i,t_{i+1},c_i) = \frac{t_i+t_{i+1}+c_i}{3}.$$

Proposition 2 states that for a given $N \geq 2$, there is an $\bar{\varepsilon}$ such that for any $\varepsilon < \bar{\varepsilon}$, a sender with $b < b_{ne}(N)$ can achieve the payoff $\pi^S(b-\varepsilon, E\,(b-\varepsilon,N))$ by using an Ellsbergian strategy. For $\varepsilon > 0$ small enough, the Ellsbergian strategy involved is given by $\left\{\{t_i(b-\varepsilon,N)\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right\}$ such that

$$c_i = \frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2} + 3\varepsilon,\; i=0,...,N-1.$$

The expected payoff of S in the corresponding Ellsbergian equilibrium is

$$\begin{aligned}
& \sum_0^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)+c_i}{3}-(\omega+b)\right)^2 d\omega \\
= & \sum_0^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2}-\omega-(b-\varepsilon)\right)^2 d\omega \\
= & \pi^S(b-\varepsilon,E(b-\varepsilon,N)) > \pi^S(b,E(b,N)).
\end{aligned}$$

Equal intervals equilibria Consider the following Ellsbergian communication strategy. For each $i = 1, ..., N$, set $t_i = \frac{i}{N}$, and set $c_i \in (t_i, t_{i+1}]$ such that $a_e^*(t_i, t_{i+1}, c_i) = \frac{t_i + t_{i+1}}{2} + b$. The state space is thus cut up into N equally sized intervals and for each such interval $(t_i, t_{i+1}]$, R responds by picking the action $\frac{t_i + t_{i+1}}{2} + b$ that S would find optimal conditional on the informational event $\omega \in (t_i, t_{i+1}]$. The key argument for examining this class of equilibria is that an equilibrium featuring this profile of strategies implements the optimal decision rule of S conditional on N different actions being taken with positive probability (we denote this rule $D(b, N)$). Conditional on S being restricted to using at most N messages, such an N-partitions *equal intervals equilibrium* is thus the optimal equilibrium among all possible cheap talk equilibria.

Proposition 4 *a) For all $b \leq \frac{1}{12}$ there is a finite $N_s(b) \geq 2$ such that there exists an equal intervals equilibrium implementing $D(b, N)$ if and only if $N \in \{2, ..., N_s(b)\}$. If $b > \frac{1}{12}$, there exists no equal intervals equilibrium implementing $D(b, N)$ for any $N \geq 2$.*

b) For all $b \leq \frac{1}{18}$ (so that $N_s(b) \geq 3$), S and R obtain a strictly higher expected payoff in an equal intervals equilibrium implementing $D(b, N')$ than in one implementing $D(b, N)$, for $N_s(b) \geq N' > N \geq 2$.

c) For all $b \leq \frac{1}{30}$, $N_s(b) > N_{ne}(b)$; For all $b \in \left(\frac{1}{30}, \frac{1}{18}\right]$, $N_s(b) = N_{ne}(b)$; For all $b \in \left(\frac{1}{18}, \frac{1}{12}\right]$, $N_s(b) < N_{ne}(b)$.

d) For all $b \leq \frac{1}{18}$, S obtains a strictly higher expected payoff in an equal intervals equilibrium implementing $D(b, N')$ than in $E(b, N)$ given $N_s(b) \geq N' \geq N \geq 2$. For all $b \leq \frac{1}{18}$, R obtains a strictly (weakly) higher expected payoff in an equal intervals equilibrium implementing $D(b, N')$ than in $E(b, N)$ given $N_s(b) \geq N' \geq N \geq 3(2)$.

Proof: see Appendix D.

Points a) and b) are reminiscent of the CS characterization for the Uniform-Quadratic case. Point a) states that there is a maximal b above which there exists no influential equal intervals equilibrium. Point b) states that finer equal intervals equilibria Pareto-dominate coarser ones. Point c) shows that for b sufficiently small, there exists an equal intervals equilibrium featuring more intervals than the finest standard equilibrium. Point d) shows that for b small enough, a given N-intervals standard equilibrium is Pareto-dominated by any equal intervals equilibrium featuring weakly more than N intervals. This is trivially true for S. As to R, this reveals that the loss implied by her distorted best responses in equal intervals equilibria is more than compensated by a more favorable profile of intervals.

Maximal ambiguity equilibria We now analyze a subclass of Ellsbergian equilibria featuring what may be described as a maximal use of simple Ellsbergian randomization (we term these *maximal ambiguity equilibria*). Given $\{t_i\}_{i=1}^{N-1}$, the involved communication strategy is constructed by setting $c_i = t_{i+1}$ for any $i = 1, ..., N$. We here compare the set of maximal ambiguity equilibria with the sets of standard and equal intervals equilibria.

Proposition 5 *a) For all $b \leq \frac{1}{3}$, there is a finite $N_m(b) \geq 2$ such that there exists an N-intervals maximal ambiguity equilibrium if and only if $N \in \{2, ..., N_m(b)\}$. For any given N, there exists at most one N-intervals maximal ambiguity equilibrium. If $b > \frac{1}{3}$, there exists no N-intervals maximal ambiguity equilibrium for any $N \geq 2$.*

b) For all $b \leq \frac{1}{4}, N_m(b) \geq N_{ne}(b)$. For all $b \in \left(\frac{1}{4}, \frac{1}{3}\right]$, $N_m(b) = 2 > N_{ne}(b) = 1$; For all $b \leq \frac{1}{12}$, $N_m(b) \geq N_s(b) + 1$.

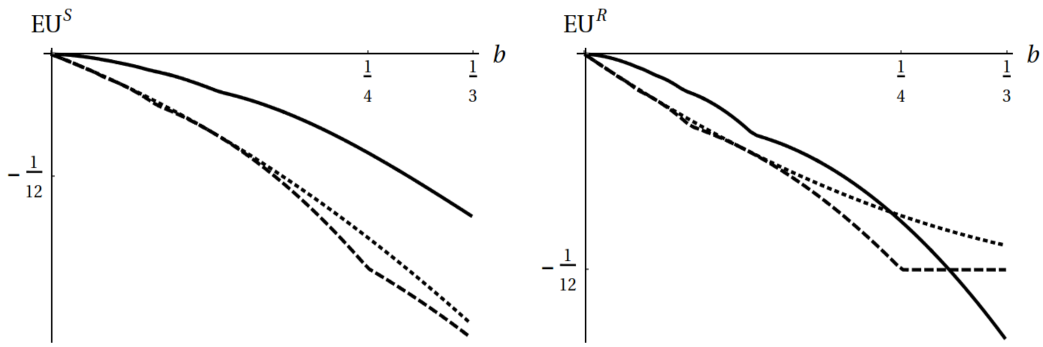
c) For all $b \leq \frac{1}{12}$, S and R obtain a strictly higher expected payoff in the maximum ambiguity equilibrium featuring $N_s(b) + 1$ intervals than in an equal intervals equilibrium implementing $D(b, N)$, for $N \in \{2, ..., N_s(b)\}$.

Proof: See Appendix E.

Point a) states that there is a maximal b above which there exists no influental maximal ambiguity equilibrium. Point b) implies together with Point c) of Proposition 3 that for $b \leq \frac{1}{18}$, the finest maximal ambiguity equilibrium is finer than the finest standard equilibrium as well as the finest equal intervals equilibrium. Point c) implies that if $b \leq \frac{1}{18}$, there exists a maximal ambiguity equilibrium that Pareto-dominates any any standard equilibrium as well as any equal intervals equilibrium.

Though influential maximal ambiguity equilibria exist for $b \leq \frac{1}{3}$ and influential standard equilibria for $b \leq \frac{1}{4}$, we do not have an analytical comparison of maximal ambiguity equilibria with standard equilibria for $b \in \left(\frac{1}{18}, \frac{1}{4}\right]$. Figure 3 and 4 complement our analytical results with a numerical analysis. We find that for $b \in \left(\frac{1}{18}, \frac{1}{4}\right]$, S and R obtain a strictly higher expected payoff in the finest maximal ambiguity equilibrium than in the finest standard equilibrium, just as for $b \leq \frac{1}{18}$.

The continuous curve in Figure 3 shows, for every b, S's expected payoff in the finest maximal ambiguity equilibrium. The dashed curve gives S's payoff in the finest standard equilibrium. The continuous and dashed curves in Figure 4 are the equivalents for R's expected payoff. The dotted curve in Figures 3 and 4 shows the payoff achieved by respectively S and R in the Pareto-optimal mediation protocol (Goltsman et al. (2009)), which is also the payoff achieved in the optimal noisy equilibrium in Board, Blume and Kawamura (2007). Note that the payoff improvement achieved by S and R through maximal ambiguity equilibria is significant. For all (all but very high) bs, S (R) prefers the best maximal ambiguity equilibrium to the best mediated communication equilibrium[4].



Figures 3 and 4

Overcoming babbling We here show that maximal ambiguity equilibria can help overcome the standard babbling equilibrium $E(b, 1)$ in a way that is beneficial to both parties.

Proposition 6 *Let $b \in (\frac{1}{4}, \frac{1}{3}]$ so that only the standard babbling equilibrium $E(b, 1)$ exists absent Ellsbergian strategies. There exists a (unique) 2-intervals maximal ambiguity equilibrium (call it $\widetilde{E}(b, 2)$). Also:*

[4]Mediated communication assumes the existence of a third party, the mediator. S communicates with the mediator who then communicates with R. The mediator can commit to a communication rule and Goltsman et al. (2009) identify the Pareto-dominant rule.

1.For any $b \in \left(\frac{1}{4}, \frac{1}{3}\right]$, S's expected payoff in $\widetilde{E}(b,2)$ is strictly larger than her expected payoff in the standard babbling equilibrium $E\left(b,1\right)$.

2.For any $b < \frac{1}{12}(1+\sqrt{6}) \simeq 0.28$, R's expected payoff in $\widetilde{E}(b,2)$ is strictly larger than her expected payoff in the standard babbling equilibrium $E\left(b,1\right)$.

Proof: See Appendix F.

6 Robustness

In what follows, we discuss the robustness of our results with respect to key restrictive assumptions of our analysis.

Endogenous choice of the urn Our game does not feature a stage at which S privately chooses an urn among a set of urns, each urn being characterized by the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ of values of ρ_1 considered possible by R. One could study a more general setup in which a continuum of such urns is present, one for each closed subset of $[0,1]$. The equilibria identified in our setup would survive. If a given strategy profile is an Ellsbergian partitional equilibrium of our simple setup, there would exist an equilibrium in the richer setup in which S picks the $[0,1]$ urn and the strategy profile of S and R is otherwise the same. S has no incentive to deviate from picking the $[0,1]$ urn because conditional on θ she randomizes between messages that yield the same expected utility.

Maximally ambiguous urn To what extent can S still use Ellsbergian randomization to shift R's response to information upwards if the unique urn available to S is characterized by $\rho_1 \in [\underline{\varepsilon}, \overline{\varepsilon}]$ with $\underline{\varepsilon}, \overline{\varepsilon} > 0$? Assuming that S uses the Ellsbergian strategy $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$, one possible approach is to identify the conditions on $\underline{\varepsilon}, \overline{\varepsilon}$ such that R's best response to m_i^A and m_i^B is still given by $a_e^*(t_i, t_{i+1}, c_i)$ as defined above. Note first that given $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ and m_i^J, for $J \in \{A, B\}$, there is a unique prior $\rho_1^J\left(t_i, t_{i+1}, c_i\right) \in (0,1)$ such that the expected utility curve $E\left[U^R(a,\omega)\middle|\, \rho_1^J(.), m_i^J\right]$ peaks at $a_e^*(t_i, t_{i+1}, c_i)$. Given $\underline{\varepsilon}, \overline{\varepsilon}$, it is easily seen that the Max-Min best response of R to m_i^J is $a_e^*(t_i, t_{i+1}, c_i)$ if and only if $\underline{\varepsilon} \leq \rho_1^J\left(t_i, t_{i+1}, c_i\right) \leq \overline{\varepsilon}$. Furthermore, it is trivially true that $\rho_1^A\left(.\right) = 1 - \rho_1^B\left(.\right)$. Given $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ and $\underline{\varepsilon}, \overline{\varepsilon}$, R's best response to m_i^A and m_i^B

will thus be $a_e^*(t_i, t_{i+1}, c_i)$ if and only if $\underline{\varepsilon} \leq \min\left\{\rho_1^A, 1-\rho_1^A\right\}$ and $\max\left\{\rho_1^A, 1-\rho_1^A\right\} \leq \overline{\varepsilon}$. In the Uniform-Quadratic setup, setting $c_i = (1-\alpha)t_i + \alpha t_{i+1}$ for each i, one obtains the simple expression

$$\rho_1^A\left(t_i, t_{i+1}, c_i\right) = \frac{\alpha^2 - 1}{2(\alpha-1)\alpha - 1}.$$

The above function equals $\frac{1}{2}$ for $\alpha = \frac{1}{2}$ and 0 for $\alpha = 1$ and it is monotonically decreasing in α. Suppose that we simply wish to ensure that given $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ and $\underline{\varepsilon}, \overline{\varepsilon}$, R's best response to m_i^A and m_i^B is $a_e^*(t_i, t_{i+1}, c_i) = a_{ne}^*(t_i, t_{i+1}) + \delta$, for δ positive and arbitrarily small. It follows from the above derivations that this can be achieved by setting $c_i = (1-\alpha)t_i + \alpha t_{i+1}$ with α arbitrarily close to $\frac{1}{2}$ if $\underline{\varepsilon} \leq \frac{\alpha^2-1}{-1+2(\alpha-1)\alpha}$ and $\overline{\varepsilon} \geq 1 - \left(\frac{\alpha^2-1}{-1+2(\alpha-1)\alpha}\right)$. The noteworthy aspect is that the bounds on $\underline{\varepsilon}$ and $\overline{\varepsilon}$ converge to $\frac{1}{2}$ for α converging to $\frac{1}{2}$, meaning that one only needs a minimally ambiguous urn to trigger a minimal upwards shift in R's best response. If on the other hand one sets $c_i = \frac{1}{3}t_i + \frac{2}{3}t_{i+1}$ for all i, our derivations imply that R's best response to m_i^A and m_i^B is $a_e^*(t_i, t_{i+1}, c_i)$ under the condition that $\underline{\varepsilon} \leq \frac{1}{3}$ and $\overline{\varepsilon} \geq \frac{2}{3}$.

The above discussion yields further insights. First, it shows that we do not need a symmetric set of priors to ensure that R's best response to m_i^A and m_i^B is identical. Second, a maximally ambiguous urn is not required for R to achieve full hedging through her best response to m_i^A and m_i^B. This is still achieved as long as R's best response remains $a_e^*(t_i, t_{i+1}, c_i)$. Third, derivations reveal that bounds on ρ_1 simply limit the range of values of c_i on $(t_i, t_{i+1}]$ for which the best response $a_e^*(t_i, t_{i+1}, c_i)$ can be achieved. A last aspect is that we assume that S knows the set of priors of R precisely. This assumption is not strictly necessary. What is essential is that whichever set of priors $[\underline{\varepsilon}, \overline{\varepsilon}]$ is used by R among those that S considers possible, the set implies the same behavior by R in response to equilibrium messages.

Strategy of R We consider a very specific decision algorithm in terms of both the belief formation rule (Prior by Prior Bayesian updating) and the decision making rule (Max-Min). Our equilibria are not robust to the use of a dynamically consistent updating rule. On the other hand, conditional on Prior by Prior Bayesian updating we would expect our main result to survive under alternative decision rules such as the α-Max-Min or the smooth ambiguity model. The key is that a shift in R's best response away from the standard best response would still be achievable through Ellsbergian randomization. Under the smooth ambiguity model, R's best response would presumably not entirely hedge her against ambiguity[5], implying that the welfare improvement result for R would have to be qualified when considering interim payoffs.

Strategy of S One could consider more complex forms of Ellsbergian randomization for example involving more than two messages on each standard interval. While we cannot exclude that this may allow to enlarge the set of implementable outcomes, two caveats are worth noting. First, it considerably simplifies the analysis to focus on strategies that imply randomization only across messages that trigger the same best response by R. Second, Ellsbergian randomization will not allow to generate a perfectly revealing equilibrium. Indeed, it only allows to shift R's response to information when applied to non-degenerate standard intervals of the state space.

7 Conclusion

This paper rationalizes ambiguous language by showing that under fairly general circumstances, a sender and an ambiguity averse receiver can both benefit from the use of ambiguous communication strategies. On the theoretical side, it remains to be explored how our insights generalize to other signaling games (costly signaling, verifiable information). On the empirical side, experimental work could help evaluate whether receivers respond to ambiguous messages as predicted by our model.

8 Appendix A

8.1 Proof of Lemma 1

The proof is constructive. Define

$$U^S(a,\omega,b) = \begin{cases} G(a-\omega-\left[\beta(\omega)+(b-1)\left(\beta'(\omega)-\beta(\omega)\right)\right]) \text{ if } b \geq 1 \\ G\left(a-\omega-\left(\beta(\omega)b^{\frac{\beta'(\omega)-\beta(\omega)}{\beta(\omega)}}\right)\right) \text{ if } b \leq 1. \end{cases}$$

[5]This is a feature of any second-order model of ambiguity. See Lang (2015).

This function clearly satisfies a), b) and c). As to d), note that $U_1^S(a,\omega,b) = G'(.)$ and that

$$U_{13}^S(a,\omega,b) = \begin{cases} -G''(a-\omega-\left[\beta(\omega)+(b-1)\left(\beta'(\omega)-\beta(\omega)\right)\right])\left[\beta'(\omega)-\beta(\omega)\right] & \text{if } b \geq 1 \\ -G''\left(a-\omega-\left(\beta(\omega)b^{\frac{\beta'(\omega)-\beta(\omega)}{\beta(\omega)}}\right)\right)\left(\beta'(\omega)-\beta(\omega)\right)b^{\frac{\beta'(\omega)-\beta(\omega)}{\beta(\omega)}} & \text{if } b \leq 1. \end{cases}$$

Given that G'' is negative everywhere and that $\beta'(\omega)-\beta(\omega) > 0$, it follows that $U_{13}^S > 0$ everywhere. Note also that $U_1^S(a,\omega,b)$ is indeed continuously differentiable in b given that $\lim_{b\to 1-} U_{13}^S(a,\omega,b) = \lim_{b\to 1+} U_{13}^S(a,\omega,b) = 1.$■

8.2 Proof of Lemma 2

In what follows, we abuse notation and denote the utility function of S by $U^S(a,\omega,\beta)$, thus explicitly referring to the bias function β. Note that β is not a scalar parameter as in the original CS setup. We have:

$$\pi^S\left(\beta, E(\beta,N)\right) = \sum_{i=0}^{N-1}\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)f\left(\omega\right)d\omega.$$

Let us define

$$\frac{d\pi^S\left(\beta,E(\beta,N)\right)}{d\beta} = \lim_{\varepsilon\to 0}\frac{\pi^S\left(\beta+\varepsilon,E(\beta+\varepsilon,N)\right)-\pi^S\left(\beta,E(\beta,N)\right)}{\varepsilon}.$$

This corresponds to the marginal effect on the payoff of S of a change in her bias function from $\beta(\omega)$ to $\beta'(\omega) = \beta(\omega) + \varepsilon$, for any ω. So let us examine:

$$
\begin{aligned}
& \frac{d\pi^S\left(\beta, E(\beta,N)\right)}{d\beta} \\
= \; & \sum_{i=0}^{N-1} \frac{d\left(\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta) f(\omega)\, d\omega\right)}{d\beta} \\
= \; & \sum_{i=0}^{N-1} \left(\begin{array}{c} \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{d\beta} f(\omega)\, d\omega \\ +U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), t_{i+1}(\beta,N), \beta) f(t_{i+1}(\beta,N)) \frac{\partial t_{i+1}(\beta,N)}{\partial \beta} \\ -U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), t_i(\beta,N), \beta) f(t_i(\beta,N)) \frac{\partial t_i(\beta,N)}{\partial \beta} \end{array} \right) \\
= \; & \sum_{i=0}^{N-1} \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{d\beta} f(\omega)\, d\omega \\
& + \sum_{i=1}^{N-1} \left(\left[\begin{array}{c} -U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), t_i(\beta,N), \beta) \\ +U^S(a^*_{ne}(t_{i-1}(\beta,N), t_i(\beta,N)), t_i(\beta,N), \beta) \end{array} \right] f(t_i(\beta,N)) \frac{\partial t_i(\beta,N)}{\partial \beta} \right) \\
& + U^S(a^*_{ne}(t_0(\beta,N), t_1(\beta,N)), t_0(\beta,N), \beta) f(t_0(\beta,N)) \frac{\partial t_0(\beta,N)}{\partial \beta} \\
& - U^S(a^*_{ne}(t_{N-1}(\beta,N), t_N(\beta,N)), t_N(\beta,N), \beta) f(t_N(\beta,N)) \frac{\partial t_N(\beta,N)}{\partial \beta}.
\end{aligned}
$$

Note first that the second line of the above expression is equal to 0 given that for every $i \in \{1, ..., N-1\}$,

$$
U^S(a^*_{ne}(t_{i-1}(\beta,N), t_i(\beta,N)), t_i(\beta,N), \beta) - U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), t_i(\beta,N), \beta) = 0.
$$

Note furthermore that by definition $\frac{\partial t_0(\beta,N)}{\partial \beta} = \frac{\partial t_N(\beta,N)}{\partial \beta} = 0$, given that $t_0(\beta,N) = 0$ and $t_N(\beta,N) = 1$. We now show that for every $i \in \{0, ..., N-1\}$,

$$
\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{d\beta} f(\omega)\, d\omega < 0.
$$

Note that for every $i \in \{0, ..., N-1\}$,

$$\begin{aligned} & \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{d\beta} f(\omega)\, d\omega \\ = & \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{\partial\beta} f(\omega)\, d\omega + \\ & \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{\partial a} f(\omega)\, d\omega \left(\begin{array}{c} \frac{\partial a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_i} \frac{\partial t_i(\beta,N)}{\partial\beta} + \\ \frac{\partial a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_{i+1}} \frac{\partial t_{i+1}(\beta,N)}{\partial\beta} \end{array} \right). \end{aligned}$$

Note first that

$$\frac{\partial U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{\partial\beta} = -\frac{\partial U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{\partial a}.$$

The above is true because we have assumed that $U^S(a,\omega,\beta(\omega)) = G(a-\omega+\beta(\omega))$ for some concave and single peaked function G. Note now that

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{\partial a} f(\omega)\, d\omega > 0. \tag{8}$$

To see this, note first that $a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))$ by definition satisfies:

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega,\beta)}{\partial a} f(\omega)\, d\omega = 0. \tag{9}$$

Second, note that we have assumed that $U^S(a,\omega,0) = U^R(a,\omega)$ and $U_{13}^S > 0$. It follows that (9) implies (8). Intuitively, R's favorite action conditional on $\omega \in (t_i(\beta,N), t_{i+1}(\beta,N)]$ is smaller than S's favoured action, thus implying that the derivative of S's expected payoff function with respect to a at the chosen a_{ne}^* must be > 0. Finally, note that

$$\begin{aligned} \frac{\partial t_i(\beta,N)}{\partial\beta} &< 0, \text{ for } i = 1, ..., N-1, \\ \frac{\partial a_{ne}^*(t_i,t_{i+1})}{\partial t_i} &> 0,\ \frac{\partial a_{ne}^*(t_i,t_{i+1})}{\partial t_{i+1}} > 0. \end{aligned}$$

The two inequalities stated in the first line follow from Lemma 4 in CS. The two inequalities stated in the second line are proved in Claim 2 below ■

9 Appendix B

9.1 Three claims

We first state three claims that hold independently of Assumption 2.

Claim 1 ***Concavity and single peakedness***

$E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]$ is a concave and single peaked function with a unique maximizer $a^*_{ne}(\underline{\omega},\overline{\omega})$.

Proof: This follows trivially from the fact that G is concave and single peaked. ■

Claim 2 ***Shift in maximum***

For any $\underline{\omega},\overline{\omega},\Delta_1,\Delta_2$ such that $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$ (with strict inequality for at least one of the two), $0 \leq \underline{\omega} < \overline{\omega} \leq 1$ and $0 \leq \underline{\omega}+\Delta_1 < \overline{\omega}+\Delta_2 \leq 1$, it holds true that $a^*_{ne}(\underline{\omega},\overline{\omega}) < a^*_{ne}(\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2)$.

Proof: We prove the statement for $\Delta_1 = 0$ and $\Delta_2 > 0$. The proof for remaining cases is similar. By the FOC defining a^*_{ne}, it is true that

$$\int_{\underline{\omega}}^{\overline{\omega}} U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega = 0 \tag{10}$$

Now, given the concavity of $\int_{\underline{\omega}}^{\overline{\omega}+\Delta_2} U^R(a,\omega)f(\omega)\,d\omega$, we simply need to prove that

$$\int_{\underline{\omega}}^{\overline{\omega}+\Delta_2} U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega > 0.$$

Note that

$$\begin{aligned} &\int_{\underline{\omega}}^{\overline{\omega}+\Delta_2} U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega \\ = &\int_{\underline{\omega}}^{\overline{\omega}} U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega + \int_{\overline{\omega}}^{\overline{\omega}+\Delta_2} U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega > 0. \end{aligned} \tag{11}$$

The first integral in (11) is equal to 0, so we simply need to prove that the second integral is strictly positive. Now, given the assumption that $U_{12}^R > 0$, note that (10) trivially implies that $U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega) > 0$ for any $\omega \geq \overline{\omega}$ (with strict inequality for $\omega > \overline{\omega}$), which in turn implies that $\int_{\overline{\omega}}^{\overline{\omega}+\Delta_2} U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega > 0.$■

Claim 3 *Single crossing condition*

1. Let $\underline{\omega},\overline{\omega},\Delta_1,\Delta_2$ be such that $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$ (with strict inequality for at least one of the two), $0 \leq \underline{\omega} < \overline{\omega} \leq 1$ and $0 \leq \underline{\omega}+\Delta_1 < \overline{\omega}+\Delta_2 \leq 1$. If a^* is such that $E\left[U^R(a^*,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right] = E\left[U^R(a^*,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]$, then for any $a > a^*$, $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right] > E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]$.

2. Let $\underline{\omega}+\Delta_1 \in (\underline{\omega},\overline{\omega}]$. There is an $a^* \in (a^*_{ne}(\underline{\omega},\underline{\omega}+\Delta_1), a^*_{ne}(\underline{\omega}+\Delta_1,\overline{\omega}))$ such that

$$E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}]\right] > E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},\underline{\omega}+\Delta_1)\right]$$

if $a > a^*$ while the inequality is reversed for $a < a^*$ and replaced by equality if $a = a^*$.

Proof: Point 1 follows since, given $U_{12} > 0$, it holds true that

$$\frac{\partial E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right]}{\partial a} > \frac{\partial E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]}{\partial a}, \forall a > a^*.$$

To see that Point 2 holds, note the following. We know by Assumption 2 that

$$E\left[U^R(a^*_{ne}(\underline{\omega},\underline{\omega}+\Delta_1),\omega)\,|\omega \in (\underline{\omega},\underline{\omega}+\Delta_1)\right] > E\left[U^R(a^*_{ne}(\underline{\omega},\underline{\omega}+\Delta_1),\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}]\right]$$

and

$$E\left[U^R(a^*_{ne}(\underline{\omega}+\Delta_1,\overline{\omega}),\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}]\right] > E\left[U^R(a^*_{ne}(\underline{\omega}+\Delta_1,\overline{\omega}),\omega)\,|\omega \in (\underline{\omega},\underline{\omega}+\Delta_1)\right].$$

Furthermore, $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},\underline{\omega}+\Delta_1)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}]\right]$ are both continuous functions of a. It follows that they must cross on the interval

$$(a^*_{ne}(\underline{\omega},\underline{\omega}+\Delta_1), a^*_{ne}(\underline{\omega}+\Delta_1,\overline{\omega}))\,.$$

■

9.2 Proof of Lemma 3

Step 1 We show in this step that it is without loss of generality to assume that the Max-Min action of R is a pure action. Consider a mixed action $\widetilde{a}$ of R given by a distribution $\widetilde{g}$ over $[0,1]$. Denote by $\overline{a}\,(\widetilde{a})$ the pure action satisfying $\overline{a}\,(\widetilde{a}) = \int_0^1 a\widetilde{g}(a)da$. Recall that the payoff function U^R is concave. It follows by Jensen's inequality that the expected payoff of $\widetilde{a}$ is weakly smaller than that of $\overline{a}\left(\widetilde{a}\right)$ for any distribution $\widehat{F}$ of the state ω, i.e.

$$\int_0^1 \left(\int_0^1 U^R(a,\omega)\widetilde{g}(a)da \right) \widehat{f}(\omega)d\omega \leq \int_0^1 U^R(\overline{a}\left(\widetilde{a}\right),\omega)\widehat{f}(\omega)d\omega.$$

R is a max-min decision maker, i.e. chooses the (possibly mixed action) a^* (given by the distribution g^*) that maximizes

$$\min_f \int_0^1 \left(\int_0^1 U^R(a,\omega)g^*(a)da \right) \widehat{f}(\omega)d\omega.$$

Suppose the optimal Max-Min action assigns positive probability to multiple pure actions, i.e. that g^* is not degenerate. We know that the pure action $\overline{a}\left(a^*\right) = \int_0^1 ag^*(a)da$ does weakly better for any distribution $\widehat{f}$ of the state. It follows that two cases are possible. Either a^* and $\overline{a}\left(a^*\right)$ are both solutions to the Max-Min problem or $\overline{a}\left(a^*\right)$ is while a^* is not. Consequently, we may without loss of generality focus on pure actions in searching for the Max-Min solution.

Step 2 This proves Point a). Assume that R has received message m_i^A. Let $E\left[U^R(a,\omega)\left|m_i^A,\rho_1\right.\right]$ denote the expected utility of action a conditional on receiving message m_i^A, assuming that S uses the communication strategy $\left(\{t_r\}_{r=1}^{N-1},\{c_r\}_{r=0}^{N-1}\right)$. We first note that for any $\rho_1 \in (0,1)$,

$$\begin{aligned} & \min\{E\left[U^R(a,\omega)|m_i^A,1\right],E\left[U^R(a,\omega)|m_i^A,0\right]\} \\ \leq \ & E\left[U^R(a,\omega)|m_i^A,\rho_1\right] \leq \max\{E\left[U^R(a,\omega)|m_i^A,1\right],E\left[U^R(a,\omega)|m_i^A,0\right]\}. \end{aligned}$$

It follows that for any given a, the set of values of ρ_1 yielding the minimum expected payoff contains either $\rho_1 = 0$ or $\rho_1 = 1$. In searching for the Max-Min action of R, we may thus without loss of generality assume that either $\rho_1 = 0$ or $\rho_1 = 1$. If $\rho_1 = 0$, m_i^A implies that $\omega \in [c_i, t_{i+1}]$. If instead $\rho_1 = 1$, m_i^A implies that $\omega \in (t_i, c_i)$.

In searching for the Max-Min best response of R, we thus only need to consider the lower envelope of

$$\left\{E\left[U^R(a,\omega)\left|\omega \in (t_i,c_i)\right.\right],E\left[U^R(a,\omega)\left|\omega \in [c_i,t_{i+1}]\right.\right]\right\}.$$

Let a^* be the unique value of a for which

$$E\left[U^R(a,\omega)\,|\omega\in(t_i,c_i)\right]=E\left[U^R(a,\omega)\,|\omega\in[c_i,t_{i+1}]\right].$$

By Claims 1-3, the lower envelope is given by $E\left[U^R(a,\omega)\,|\omega\in[c_i,t_{i+1}]\right]$ for $a\leq a^*$and by $E\left[U^R(a,\omega)\,|\omega\in(t_i,c_i)\right]$ for $a>a^*$. The lower envelope is strictly increasing in a for $a<a^*$ and strictly decreasing in a for $a>a^*$. It thus has a unique maximum at a^*.

Assume instead that R has received m_i^B. If $\rho_1=1$, m_i^B implies that $\omega\in[c_i,t_{i+1}]$. If instead $\rho_1=0$, m_i^B implies that $\omega\in(t_i,c_i)$. By the same argument as above, it follows that R's Max-Min action is given by a^* as defined above.

Step 3 This proves the first part of Point b), i.e. that $a_e^*(t_i,t_{i+1},c_i)$ is a continuous and strictly increasing function of c_i. We simply state and prove the following statement:

Let $0\leq\underline{\omega}<\overline{\omega}\leq 1$. Let $a^*(c)$ be the unique value a such that

$$E\left[U^R(a^*(c),\omega)\,|\omega\in(\underline{\omega},c)\right]=E\left[U^R(a^*(c),\omega)\,|\omega\in[c,\overline{\omega}]\right].\tag{12}$$

It holds true that $a^*(c)$ is continuous and strictly increasing in c on $[\underline{\omega},\overline{\omega}]$.

Let $\underline{\omega}\leq c<c'\leq\overline{\omega}$. Consider the three functions given by $E\left[U^R(a,\omega)\,|\omega\in[(\underline{\omega},c)\right]$, $E\left[U^R(a,\omega)\,|\omega\in[c,c')\right]$ and $E\left[U^R(a,\omega)\,|\omega\in[c',\overline{\omega}]\right]$. We know that $a_{ne}^*(\underline{\omega},c)<a_{ne}^*(c,c')<a_{ne}^*(c',\overline{\omega})$. Furthermore, by Claim 3 the unique crossing point a_1 of

$$E\left[U^R(a,\omega)\,|\omega\in(\underline{\omega},c)\right]\text{ and }E\left[U^R(a,\omega)\omega\in[c,c')\right]$$

belongs to $(a_{ne}^*(\underline{\omega},c),a_{ne}^*(c,c'))$. Similarly, by Claim 3 the unique crossing point a_3 of

$$E\left[U^R(a,\omega)\,\left|\omega\in[c,c')\right.\right]\text{ and }E\left[U^R(a,\omega)\,\left|\omega\in[c',\overline{\omega}]\right.\right]$$

belongs to $(a_{ne}^*(c,c'),a_{ne}^*(c',\overline{\omega}))$. It also follows that the unique crossing point a_2 of

$$E\left[U^R(a,\omega)\,|\omega\in(\underline{\omega},c)\right]\text{ and }E\left[U^R(a,\omega)\,|\omega\in[c',\overline{\omega}]\right]$$

belongs to (a_1,a_3). We thus have $a_1<a_2<a_3$.

Now, let us first compare $E\left[U^R(a,\omega)\,|\omega\in(\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega\in[c,\overline{\omega}]\right]$ and show that they have a unique crossing point at some $a^*\in[a_1,a_2)$. Note that for every a, there is some $\alpha\in(0,1)$ such that

$$E\left[U^R(a,\omega)\,|\omega\in[c,\overline{\omega}]\right]=\alpha E\left[U^R(a,\omega)\,\left|\omega\in[c,c')\right.\right]+(1-\alpha)E\left[U^R(a,\omega)\,\left|\omega\in[c',\overline{\omega}]\right.\right].$$

We know that for any $a < a_1$,

$$\max\left\{E\left[U^R(a,\omega)\,|\omega \in [c,c')\right], E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]\right\} < E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right].$$

It follows that for $a < a_1$, $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] > E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$. We also know that for any $a \geq a_2$,

$$\begin{aligned} E\left[U^R(a,\omega)\,|\omega \in [c,c')\right] &> E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] \\ \text{and } E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right] &\geq E\left[U^R(a,\omega)\,|\omega \in \underline{\omega},c\right]. \end{aligned}$$

It follows that for $a \geq a_2$, $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] < E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$. We may conclude that $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$ cross somewhere on $[a_1, a_2)$.

Let us now compare $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c')\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$ and show that they have a unique crossing point at some $a^* \in (a_2, a_3]$. Note that for every a, there is some $\widetilde{\alpha} \in (0,1)$ such that

$$E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c')\right] = \widetilde{\alpha}E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] + (1-\widetilde{\alpha})E\left[U^R(a,\omega)\,|\omega \in [c,c')\right].$$

We know that for any $a \leq a_2$,

$$\begin{aligned} E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] &\geq E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right] \\ \text{and } E\left[U^R(a,\omega)\,|\omega \in [c,c')\right] &> E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]. \end{aligned}$$

It follows that for $a \leq a_2$, $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right] > E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$. We also know that for any $a > a_3$,

$$\max\left\{E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c)\right], E\left[U^R(a,\omega)\,|\omega \in [c,c')\right]\right\} < E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right].$$

It follows that for $a > a_3$, $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right] < E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$. We may conclude that $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$ cross somewhere on $(a_2, a_3]$.

Having now proved that $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$ cross somewhere on $[a_1, a_2)$ while $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$ cross somewhere on $(a_2, a_3]$, it follows that $a^*(c) < a^*(c')$. The continuity of $a^*(c)$ in c follows from the continuity of $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}, c)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c, \overline{\omega}]\right]$ in c.

Step 4 This proves the double inequality contained in Point b). Finally, to see that $a^*_{ne}(t_i, t_{i+1}) < a^*_e(t_i, t_{i+1}, t_{i+1})$, note that it follows immediately from Claims 1-3 that the value of a ensuring equality of $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega}, \overline{\omega}]\right]$ and $E\left[U^R(a,\omega)\,|\omega = \overline{\omega}\right]$ is strictly larger than $a^*_{ne}(\underline{\omega}, \overline{\omega})$. If similarly follows that the value of a ensuring equality of $E\left[U^R(a,\omega)\,|\omega = \underline{\omega}\right]$ and $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega}, \overline{\omega}]\right]$ is strictly smaller than $a^*_{ne}(\underline{\omega}, \overline{\omega})$.■

10 Appendix C

We here prove Proposition 2.

Step 1 This step proves Point a). In what follows, as in Appendix A, we abuse notation and denote the utility function of S by $U^S(a, \omega, \beta)$, thus explicitly referring to the bias function β. Note that β is not a scalar parameter as in the original CS setup.

Assume that β, N such that $E(\beta, N)$ exists and is non-degenerate, i.e. $0 < t_1(\beta, N) < ... < t_{N-1}(\beta, N) < 1$. First, Lemma 3 (Part b)) implies that for any given β, N such that $0 < t_1(\beta, N) < ... < t_{N-1}(\beta, N) < 1$, there is some maximal $\overline{\delta}(\beta, N) > 0$ such that for any $\delta \leq \overline{\delta}(\beta, N)$, one can pick a profile $\{c_i\}_{i=0}^{N-1}$ satisfying $c_i \in (t_i(\beta, N), t_{i+1}(\beta, N)]$ such that for every $i \in \{0, ..., N-1\}$:

$$a^*_e(t_i(\beta, N), t_{i+1}(\beta, N), c_i) = a^*_{ne}(t_i(\beta, N), t_{i+1}(\beta, N)) + \delta.$$

Note that

$$\overline{\delta}(\beta, N) = \min_{i=0,...,N-1} a^*_e(t_i(\beta, N), t_{i+1}(\beta, N), t_{i+1}(\beta, N)) - a^*_{ne}(t_i(\beta, N), t_{i+1}(\beta, N)).$$

Second, note that if β, N is such that $0 < t_1(\beta, N) < ... < t_{N-1}(\beta, N) < 1$, then there is some $\overline{\varepsilon}(\beta, N) > 0$ such that for any $\varepsilon \leq \overline{\varepsilon}(\beta, N)$, it holds true that $0 < t_1(\beta - \varepsilon, N) < ... < t_{N-1}(\beta - \varepsilon, N) < 1$. It follows by Lemma 3 (Part b)) that for any $\varepsilon \leq \overline{\varepsilon}(\beta, N)$, $\overline{\delta}(\beta - \varepsilon, N) > 0$. Finally, note that for any $\varepsilon \leq \overline{\varepsilon}(\beta, N)$, $\overline{\delta}(\beta - \varepsilon, N)$ is continuous in ε. To see this, recall that

$$\overline{\delta}(\beta - \varepsilon, N) = \min_{i=0,...,N-1} a^*_e(t_i(\beta - \varepsilon, N), t_{i+1}(\beta - \varepsilon, N), t_{i+1}(\beta - \varepsilon, N)) - a^*_{ne}(t_i(\beta - \varepsilon, N), t_{i+1}(\beta - \varepsilon, N))$$

and simply note that $a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))$ and $a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))$ are continuous in ε for the following reasons. First, $E\left[U^R(a,\omega)\,|\omega\in[t_i,t_{i+1})\right]$ is continuous in t_i and t_{i+1} while $E\left[U^R(a,\omega)\,|\omega=t_{i+1}\right]$ is in continuous t_{i+1}. Second, $t_i(\beta-\varepsilon,N)$ and $t_{i+1}(\beta-\varepsilon,N)$ are continuous in ε.

We may conclude that there exists an $\bar{\bar{\varepsilon}}(\beta,N)\in(0,\bar{\varepsilon}(\beta,N)]$ such that for any $\varepsilon\leq\bar{\bar{\varepsilon}}(\beta,N)$, it holds true that $\bar{\delta}(\beta-\varepsilon,N)\geq\varepsilon$. For any $\varepsilon\in(0,\bar{\bar{\varepsilon}}(\beta,N)]$, there thus exists some profile $\{c_i\}_{i=0}^{N-1}$ satisfying $c_i\in(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)]$ such that for every $i\in\{0,...,N-1\}$:

$$a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i)=a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon.$$

For any $\varepsilon\in(0,\bar{\bar{\varepsilon}}(\beta,N)]$ and such a profile $\{c_i\}_{i=0}^{N-1}$, note that for every $i\in\{0,...,N-1\}$ and ω :

$$\begin{aligned}&U^S\left(a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i),\omega,\beta\right)\\ =\;&U^S\left(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)),\omega,\beta-\varepsilon\right).\end{aligned}$$

For any $\varepsilon\in(0,\bar{\bar{\varepsilon}}(\beta,N)]$, given that for every $i\in\{1,...,N-1\}$

$$\begin{aligned}&U^S\left(a_{ne}^*(t_{i-1}(\beta-\varepsilon,N),t_i(\beta-\varepsilon,N)),t_i(\beta-\varepsilon,N),\beta-\varepsilon\right)\\ =\;&U^S\left(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)),t_i(\beta-\varepsilon,N),\beta-\varepsilon\right),\end{aligned}$$

as implied by the existence of the standard equilibrium $E(\beta-\varepsilon,N)$ for a sender bias given by $\beta-\varepsilon$, it thus follows that for every $i\in\{1,...,N-1\}$,

$$\begin{aligned}&U^S\left(a_e^*(t_{i-1}(\beta-\varepsilon,N),t_i(\beta-\varepsilon,N),c_{i-1,i}),t_i(\beta-\varepsilon,N),\beta\right)\\ =\;&U^S\left(a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i),t_i(\beta-\varepsilon,N),\beta\right),\end{aligned}$$

which implies that the equilibrium $\widetilde{E}$ exists.

Step 2 This proves Point b). The expected payoff of S in $\widetilde{E}$ is given by:

$$\sum_{i=0}^{N-1}\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)}U^S\left(a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i),\omega,\beta\right)f(\omega)d\omega$$

Recall that in $\widetilde{E}$, the profile $\{c_i\}_{i=0}^{N-1}$ is picked such that for every $i\in\{0,...,N-1\}$,

$$a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i)=a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon.$$

Combining the above with the fact that $U^S(a+\varepsilon,\omega,\beta) = U^S(a,\omega,\beta-\varepsilon)$, the expected payoff of S in $\widetilde{E}$ can thus be rewritten as

$$\sum_{i=0}^{N-1} \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^S\left(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)),\omega,\beta-\varepsilon\right) f(\omega)d\omega.$$

The above expression is equal to $\pi^S(\beta-\varepsilon, E(\beta-\varepsilon,N))$ by definition of the standard equilibrium $E(\beta-\varepsilon,N)$. Finally, we know by Lemma 2 that

$$\pi^S(\beta-\varepsilon, E(\beta-\varepsilon,N)) > \pi^S(\beta, E(\beta,N)).$$

Step 3 This proves Point c). Note first that

$$\pi^R\left(\widetilde{E}\right) = \sum_{i=0}^{N-1} \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega) f(\omega)\,d\omega.$$

We have:

$$\frac{d\pi^R\left(\widetilde{E}\right)}{d\varepsilon} = \left(\sum_{i=0}^{N-1} \frac{d\left(\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega) f(\omega)\,d\omega\right)}{d\varepsilon}\right).$$

By Leibniz rule, the above can be rewritten as

$$\sum_{i=0}^{N-1}\left(\begin{array}{c} \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} \frac{dU^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega) f(\omega)}{d\varepsilon} d\omega \\ +U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,t_{i+1}(\beta-\varepsilon,N)) f(t_{i+1}(\beta-\varepsilon,N)) \frac{dt_{i+1}(\beta-\varepsilon,N)}{d\varepsilon} \\ -U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,t_i(\beta-\varepsilon,N)) f(t_i(\beta-\varepsilon,N)) \frac{dt_i(\beta-\varepsilon,N)}{d\varepsilon} \end{array}\right).$$

Note that:

$$\begin{aligned} & \left.\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} \frac{dU^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega) f(\omega)}{d\varepsilon} d\omega\right|_{\varepsilon=0} \\ = & \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} \frac{\partial U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega)}{\partial a} f(\omega)\,d\omega \\ & \times\left(1-\frac{\partial a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_i}\frac{\partial t_i(\beta,N)}{\partial \beta}-\frac{\partial a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_{i+1}}\frac{\partial t_{i+1}(\beta,N)}{\partial \beta}\right). \end{aligned}$$

Given the FOCs characterizing $a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))$, it follows that for every $i \in \{0,...,N-1\}$,

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega)}{\partial a} f(\omega)\,d\omega = 0$$

It follows that

$$\left.\frac{d\pi^R\left(\widetilde{E}\right)}{d\varepsilon}\right|_{\varepsilon=0} = \sum_{i=0}^{N-1}\left(\begin{array}{c} -U^R(a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N)),t_{i+1}(\beta,N))f\left(t_{i+1}(\beta,N)\right)\frac{\partial t_{i+1}(\beta,N)}{\partial\beta} \\ +U^R(a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N))f\left(t_i(\beta,N)\right)\frac{\partial t_i(\beta,N)}{\partial\beta} \end{array}\right)$$
$$= \sum_{i=1}^{N-1}\left[\begin{array}{c} -U^R(a^*_{ne}(t_{i-1}(\beta,N),t_i(\beta,N)),t_i(\beta,N)) \\ +U^R(a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N)) \end{array}\right] f\left(t_i(\beta,N)\right)\frac{\partial t_i(\beta,N)}{\partial\beta}$$

Given Condition **M**, $\frac{\partial t_i(\beta,N)}{\partial\beta} < 0$ for every $i \in \{1,...,N-1\}$. Furthermore, for every $i \in \{1,...,N-1\}$,

$$\begin{aligned} U^R(a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N)) - U^R(a^*_{ne}(t_{i-1}(\beta,N),t_i(\beta,N)),t_i(\beta,N)) &< \\ U^S(a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N),\beta) - U^S(a^*_{ne}(t_{i-1}(\beta,N),t_i(\beta,N)),t_i(\beta,N),\beta) &= 0. \end{aligned}$$

The equality appearing on the second line holds true by definition because $\{t_i(\beta,N)\}_{i=1}^{N}$ is a standard equilibrium partitional communication strategy.

The inequality appearing on the first line holds true by the following argument. By Lemma 1, there is a function $\widetilde{U}^S(a,\omega,b)$, where b is a scalar parameter, such that 1) $\widetilde{U}^S(a,\omega,1) = U^S(a,\omega,\beta)$, 2) $\widetilde{U}^S(a,\omega,0) = U^R(a,\omega)$ and 3) $\widetilde{U}^S_{13}(a,\omega,b)$ is strictly positive everywhere. Using furthermore the fact that $a^*_{ne}(t_{i-1}(\beta,N),t_i(\beta,N)) < a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N))$, the inequality follows. ■

11 Appendix D

We here prove Proposition 4.

Step 1 This proves Point a). First, set $\{t_i\}_{i=1}^{N-1}$ by letting $t_i = \frac{i}{N}$ for $i = 1,...,N$. Second, let c_i satisfy

$$\frac{t_i + t_{i+1} + c_i}{3} = \frac{t_i + t_{i+1}}{2} + b,\ \forall i = 0,...,N-1, \tag{13}$$

so that $a^*_e(t_i,t_{i+1},c_i) = a^*_{ne}(t_i,t_{i+1}) + b$, $\forall i = 0,...,N-1$. Condition (13) is feasible iff given any $i \in \{0,...,N-1\}$,

$$\frac{t_i + t_{i+1}}{2} + b \leq \frac{t_i + 2t_{i+1}}{3}$$

which is equivalent to $b \leq \frac{t_{i+1}-t_i}{6}$. Using the fact that by definition $t_{i+1} - t_i = \frac{1}{N}$, this condition simplifies to $b \leq \frac{1}{6N}$.

Assuming that the above condition is satisfied, the constructed strategy constitutes an equilibrium iff for any $i \in \{1, ..., N-1\}$,

$$-\left(\frac{t_{i-1}+t_i+c_{i-1}}{3} - t_i - b\right)^2 = -\left(\frac{t_i + t_{i+1} + c_i}{3} - t_i - b\right)^2.$$

Using (13) as well as the fact that $t_i = \frac{i}{N}$ for $i = 1, ..., N$, the above is equivalent to

$$\left(\left(\frac{2i+1}{2N} + b\right) - \frac{i}{N} - b\right)^2 = \left(\left(\frac{2i-1}{2N} + b\right) - \frac{i}{N} - b\right)^2,$$

which simplifies to $\left(\frac{1}{2N}\right)^2 = \left(-\frac{1}{2N}\right)^2$, which is always true. The obtained condition $6Nb \leq 1$ means that $\forall b$, there exists an equal intervals equilibrium implementing $D(b, N)$ if and only if $N \leq N_s(b) = \left\langle \frac{1}{6b} \right\rangle$.

Step 2 This proves Point b). As a preliminary comment, note that we will show in the next step that for $b \leq \frac{1}{18}$, $N_s(b) \geq 3$. Note that

$$\begin{aligned}
\pi^R(D(b,N)) &= -\sum_{i=1}^{N} \int_{\frac{i-1}{N}}^{\frac{i}{N}} \left(\omega - \left(\frac{\frac{i-1}{N} + \frac{i}{N}}{2} + b\right)\right)^2 d\omega \\
&= -\sum_{i=1}^{N} \left(-\frac{1}{12}\left(\frac{i-1}{N} - \frac{i}{N}\right)^3 - \left(\frac{i-1}{N} - \frac{i}{N}\right) b^2\right) = -b^2 - \frac{1}{12N^2}.
\end{aligned}$$

It can similarly be shown that $\pi^S(b, D(b,N)) = -\frac{1}{12N^2}$. It is immediate that $\pi^S(b, D(b,N))$ and $\pi^R(D(b,N))$ are increasing in N.

Step 3 This proves Point c).Note that for $b \leq \frac{1}{4} - \frac{1}{12}\sqrt{7} \in \left(\frac{1}{40}, \frac{1}{30}\right)$, it holds true that

$$\frac{1}{6b} - \left(\frac{1}{2b}\left(b + \sqrt{b\,(b+2)}\right)\right) \geq 1.$$

It follows that for $b \leq \frac{1}{40}$, $N_s(b) > N_{ne}(b)$. For $b \in \left(\frac{1}{40}, \frac{1}{30}\right]$, $N_s(b) = 5$ while $N_{ne}(b) = 4$. For $b \in \left(\frac{1}{30}, \frac{1}{24}\right]$, note that $N_{ne}(b) = N_s(b) = 4$. For $b \in \left(\frac{1}{24}, \frac{1}{18}\right]$, note that $N_s(b) = N_{ne}(b) = 3$. For $b \in \left(\frac{1}{18}, \frac{1}{12}\right]$, note that $N_{ne}(b) = 3$ while $N_s(b) = 2$.

Step 4 This proves Point d). It is a priori clear that for any N and $N' \geq N$, $\pi^S(b, D(b, N')) > \pi^S(b, E(b, N))$. Recall that $\pi^R(D(b, N)) = -b^2 - \frac{1}{12N^2}$ and that $\pi^R(D(b, N))$ is thus increasing in N. On the other hand, $\pi^R(E(b, N))$ is $-\frac{1}{12N^2} - \frac{b^2(N^2-1)}{3}$. So

$$\begin{aligned} & \pi^R(D(b,N)) - \pi^R(E(b,N)) \\ = \quad & -b^2 - \frac{1}{12N^2} - \left(-\frac{1}{12N^2} - \frac{b^2(N^2-1)}{3}\right) = \frac{1}{3}b^2\left(N^2 - 4\right). \end{aligned}$$

Note that the above expression is weakly (strictly) positive for any $N \geq (>)2$. It follows that R always strictly gains from the transition from $E\,(b, N)$ to an equilibrium implementing $D(b, N')$, given $N' \geq N \geq 3$ while he instead weakly gains from the transition from $E\,(b, 2)$ to an equilibrium implementing $D(b, 2)$. ■

12 Appendix E

We here prove Proposition 5.

12.1 Points a) and b)

Step 1 This proves Point a). Let $0 < t_1 < ... < t_N \leq 1$ and set, for every i, $c_i = t_{i+1}$. For such an Ellsbergian strategy to be incentive compatible, we need that $\forall\, i = 1, ..., N-1$,

$$\left(\frac{t_i + 2t_{i+1}}{3} - t_i - b\right)^2 = \left(\frac{t_{i-1} + 2t_i}{3} - t_i - b\right)^2,$$

which is equivalent to

$$t_{i+1} = \frac{3}{2}t_i - \frac{1}{2}t_{i-1} + 3b. \tag{14}$$

Solving the above linear difference equation, we obtain a unique solution parameterized by t_1:

$$t_i = 2^{1-i}\left[6b - 3b2^{i+1} + 3bi2^i - t_1 + 2^i t_1\right]. \tag{15}$$

Now, pick an N. Setting $t_N = 1$, solve for the (unique) implied t_1 which is given by

$$\widetilde{t}_1(b, N) = -\frac{\left(12b - 12\left(2^N\right)b - 2^N + 6\left(2^N\right)Nb\right)}{2\left(2^N\right) - 2}$$

which is a decreasing function of N and b. If this value belongs to $(0,1)$, there is a unique threshold profile $\{\widetilde{t}_r(b,N)\}_{r=1}^{N-1}$ constituting an N-intervals maximal ambiguity equilibrium. Thresholds satisfy, for $i=1,...,N$,

$$\widetilde{t}_i(b,N) = 2^{1-i}\left(6b - 3b2^{i+1} + 3b(i)2^i + (-1+2^i)\left(-\frac{(12b - 12\,(2^N)\,b - 2^N + 6\,(2^N)\,Nb)}{2\,(2^N) - 2}\right)\right).$$

We may now look for the maximal value of b compatible with the existence of an N-intervals maximum ambiguity equilibrium. Call this $b_m(N)$. To find it, solve for b such that $\widetilde{t}_1(b,N) = 0$. We find

$$b_m(N) = \frac{1}{2^{1-N}\left[6 - 3\,(2^{N+1}) + 3N2^N\right]}.$$

Similarly, $N_m(b)$ is the largest positive integer such that $\widetilde{t}_1(b,N) \geq 0$.

Step 2 This proves Point b). Note that

$$\frac{b_m(N)}{b_{ne}(N)} = \frac{2^N N}{2 + 2^N(N-1)} > 1 \text{ for } N \geq 2.$$

Thus, $b_m(N) > b_{ne}(N)\,, \forall N \geq 2$. It follows that $\forall\ b \leq b_m(2)$, there exists a maximal ambiguity equilibrium as fine as the finest standard equilibrium. Note that $b_m(2) = \frac{1}{3}$ while $b_{ne}(2) = \frac{1}{4}$. Note also that

$$b_m(N+1) - b_s(N) = \frac{1}{6N}\frac{2^N - 1}{2^N N - 2^N + 1} > 0\,, \forall N \geq 2.$$

12.2 Point c)

Outline Step 1 proves the statement of Point c) for S. It states three facts and shows that these together imply the statement. In steps 2-5, we prove the three facts invoked in step 1. Step 6 is an equivalent of step 1 that proves the statement of Point c) for R.

Step 1 Given $b \leq b_s(N)$, we slightly abuse notation and denote by respectively $\pi^S(D(b,N))$ and $\pi^R(D(b,N))$ the expected payoff of respectively S and R in an equilibrium implementing $D(b,N)$. Given $b \leq b_m(N)$, we similarly denote by respectively $\pi^S(M(b,N))$ and $\pi^R(M(b,N))$ the expected payoff of S and R in the unique N-intervals maximal ambiguity equilibrium. Note that $\forall N \geq 2$, $\pi^S(D(b,N)) = -\frac{1}{12N^2}$ which is constant in b and note the following three further facts:

Fact 1: $\forall N \geq 2$, $\pi^S(M(b_s(N), N)) = \pi^S(D(b_s(N), N))$.

Fact 2: $\forall N \geq 2$, $\pi^S(M(b_s(N), N+1)) > \pi^S(D(b_s(N), N))$.

Fact 3: $\forall N \geq 2$ and $b \in (b_s(N+1), b_s(N)]$, $\pi^S(M(b, N+1))$ is strictly decreasing in b.

Consider thus any $b \leq b_s(2)$. We know that there is an $N \geq 2$ such that $b \in (b_s(N+1), b_s(N)]$ and $b \leq b_m(N+1)$. Furthermore, it follows from Facts 1, 2 and 3 that for this N,

$$\pi^S(M(b, N+1)) > \pi^S(D(b, N)).$$

We simply prove Facts 1, 2 and 3 in what follows.

Step 2 This step proves Fact 1. Note that the condition defining $b_s(N)$ (recalling that we set $t_i = \frac{i}{N}$ in equal intervals equilibria) is

$$\frac{t_i + t_{i+1}}{2} + b = \frac{t_i + 2t_{i+1}}{3}.$$

In other words, for the highest possible value of b compatible with the existence of an N-intervals equal intervals equilibrium, the unique N-intervals equal intervals equilibrium is actually the unique N-intervals maximum ambiguity equilibrium.

Step 3 This step proves Fact 2. Simply note that

$$\begin{aligned} & \pi^S(M(b_s(N), N+1)) - \pi^S(D(b_s(N), N)) \\ = & \ \frac{4(2^N - 1)(17\left(2^N\right) - 13)}{189(2^{N+1} - 1)N^3} > 0. \end{aligned}$$

Step 4 This step proves Fact 3. Let $\left\{\widetilde{t}_i(b, N)\right\}_{i=1}^{N-1}$ be the profile of thresholds characterizing the unique Ellsbergian N-intervals maximal ambiguity equilibrium. Using the explicit formula derived for thresholds, note that for $i = 1, ..., N$,

$$\frac{\partial \widetilde{t}_i(b, N)}{\partial b} = \frac{6\left(2^N N(1 - 2^i) - 2^i i(1 - 2^N)\right)}{2^i\left(2^N - 1\right)} < 0.$$

Let us now explicitly consider the derivative $\frac{\partial \pi^S(M(b,N))}{\partial b}$. We have:

$$\pi^S(M(b, N)) = \sum_{i=0}^{N-1} \int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} U^S(a_e^*(\widetilde{t}_i(b, N), \widetilde{t}_{i+1}(b, N), \widetilde{t}_{i+1}(b, N)), \omega, b) f(\omega)\, d\omega.$$

The closed form expression for the above expected payoff is complex so that it is more convenient to work with the general formula. Thus,

$$\begin{aligned}
&\frac{d\pi^S\left(M(b,N)\right)}{db}\\
&= \sum_{i=0}^{N-1} \frac{d\left(\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)f\left(\omega\right)d\omega\right)}{db}\\
&= \sum_{i=0}^{N-1}\left(\begin{array}{c}
\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db} f\left(\omega\right)d\omega\\
+U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\widetilde{t}_{i+1}(b,N),b)f\left(\widetilde{t}_{i+1}(b,N)\right)\frac{\partial\widetilde{t}_{i+1}(b,N)}{\partial b}\\
-U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\widetilde{t}_i(b,N),b)f\left(\widetilde{t}_i(b,N)\right)\frac{\partial\widetilde{t}_i(b,N)}{\partial b}
\end{array}\right)\\
&= \sum_{i=0}^{N-1}\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db} f\left(\omega\right)d\omega,
\end{aligned}$$

In order to obtain the last equality, we use the fact that $\forall i\in\{1,...,N-1\}$,

$$U^S(a_e^*(\widetilde{t}_{i-1}(b,N),\widetilde{t}_i(b,N),\widetilde{t}_i(b,N)),\widetilde{t}_i(b,N),b)-U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\widetilde{t}_i(b,N),b)=0,$$

as well as $\widetilde{t}_0(b,N)=0$ and $\widetilde{t}_N(b,N)=1$. We now show that for every $i\in\{0,...,N-1\}$,

$$\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db} f\left(\omega\right)d\omega<0.$$

Note that for every $i\in\{0,...,N-1\}$,

$$\begin{aligned}
&\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db} f\left(\omega\right)d\omega\\
=\;&\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial b} f\left(\omega\right)d\omega+\\
&\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial a} f\left(\omega\right)d\omega\\
&\times\left(\begin{array}{c}
\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial\widetilde{t}_i}\frac{\partial\widetilde{t}_i(b,N)}{\partial b}+\\
\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial\widetilde{t}_{i+1}}\frac{\partial\widetilde{t}_{i+1}(b,N)}{\partial b}+\\
\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial c_i}\frac{\partial\widetilde{t}_{i+1}(b,N)}{\partial b}
\end{array}\right).
\end{aligned}$$

Note first that

$$\frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial b} = -\frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial a}.$$

The above is true because we have assumed that $U^S(a,\omega,b) = -(a-(\omega+b))^2$. Note now that

$$\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial a} f(\omega)\, d\omega > 0. \tag{16}$$

Call this Fact A. We prove this fact in step 5. Finally, note that

$$\begin{aligned}
\frac{\partial \widetilde{t}_i(b,N)}{\partial b} &< 0, \ \frac{\partial \widetilde{t}_{i+1}(b,N)}{\partial b} < 0, \\
\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial \widetilde{t}_i} &> 0, \ \frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial \widetilde{t}_{i+1}} > 0 \\
\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial c_i} &> 0.
\end{aligned}$$

The inequalities on the first line were shown to be true in the beginning of step 4. The inequality on the third line is proved in Point b) of Lemma 3. The inequalities on the second line can be proved along similar lines as Point b) of Lemma 3.

Step 5 This proves Fact A. Note that given $[x,y] \subseteq [0,1]$ and some action a,

$$\frac{\partial\left(-\int_x^y (a-(\omega+b))^2\right)}{\partial a} = 2\,(y-x)\left(\left(\frac{x+y}{2}+b\right)-a\right).$$

The above expression thus has the same sign as $\left(\frac{x+y}{2}+b\right)-a$. Recall furthermore that

$$a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)) = \frac{\widetilde{t}_i(b,N)+2\widetilde{t}_{i+1}(b,N)}{3}.$$

Now, note that for $i = 0,...,N-1$,

$$\left(\frac{\widetilde{t}_i(b,N)+\widetilde{t}_{i+1}(b,N)}{2}+b\right)-\left(\frac{\widetilde{t}_i(b,N)+2\widetilde{t}_{i+1}(b,N)}{3}\right) = \frac{1}{12}2^{N-i}\frac{6Nb-1}{2^N-1},$$

which is equal to 0 if $b = b_s(N)$ and positive if $b > b_s(N)$.

Step 6 We now prove the statement of Point c) for R. Note now the following facts:

Fact I: For $N \geq 2$, $\pi^R(D(b,N)) = -\frac{1}{12N^2} - b^2$ and is thus decreasing in b.

Fact II: $\forall N \geq 2$ and $b \in (b_s(N+1), b_s(N)]$, $\pi^R(M(b,N+1))$ is strictly decreasing in b.

Fact III: $\forall N \geq 2$, $\pi^R(M(b_s(N),N+1)) > \pi^R(D(b_s(N+1),N))$.

Consider thus any $b \leq b_s(2)$. We know that there is a N such that $b \in (b_s(N+1), b_s(N)]$ and $b \leq b_m(N+1)$. Furthermore, it follows from facts I, II and III that $\pi^R(M(b,N+1)) > \pi^R(D(b,N))$. The (algebraically tedious but conceptually simple) proof of Fact II is omitted. To see that Fact III holds, note that

$$\pi^R(M(b_s(N),N+1)) - \pi^R(D(b_s(N+1),N)) = \frac{\frac{12}{(2^{1+N}-1)^2} - \frac{44}{(2^{1+N}-1)} + \frac{32+3N(19+6N)}{(1+N)^2}}{252N^3} > 0.$$

■

13 Appendix F

We here prove Proposition 6. We know from the proof of Proposition 5 that $\widetilde{E}(b,2)$ exists if and only if $b \leq \frac{1}{3}$. In $\widetilde{E}(b,2)$, t_1 solves:

$$\frac{t_1+2}{3} - t_1 - b = t_1 + b - \frac{2t_1}{3} \Leftrightarrow t_1 = \frac{2}{3} - 2b.$$

$\pi^S\left(b, \widetilde{E}(b,2)\right)$ is thus:

$$-\int_0^{\frac{2}{3}-2b} \left(\left(\frac{2\left(\frac{2}{3}-2b\right)}{3}\right) - (\omega+b)\right)^2 d\omega - \int_{\frac{2}{3}-2b}^{1} \left(\left(\frac{\frac{2}{3}-2b+2}{3}\right) - (\omega+b)\right)^2 d\omega,$$

which is equal to $\frac{8}{3}b^3 - \frac{25}{9}b^2 + \frac{11}{27}b - \frac{1}{27}$. This expression is strictly larger than $\pi^S(b, E(b,1)) = -b^2 - \frac{1}{12}, \forall b \in \left(\frac{1}{4}, \frac{1}{3}\right]$. We have $\pi^R\left(\widetilde{E}(b,2)\right) = -\frac{1}{27} + \frac{2}{9}b - \frac{4}{3}b^2$ while $\pi^R(E(b,1)) = -\frac{1}{12}$. The first expression is larger than the second iff $b \leq \frac{1}{12}(1+\sqrt{6}) \simeq 0.28$.■

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